1 4th Year Materials Engineering

Mechanics of Composite Materials – Lecture 6

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2 Revision

Composite properties based on properties of components

$$
\begin{aligned}
E_1 &= E_f\phi_f + E_m(1-\phi_f) \approx E_f\phi_f \\
E_2 &= \frac{E_f E_m}{E_f\phi_m + E_m\phi_f} \approx \frac{E_m}{(1-\phi_f)} \\
\nu_{12} &= \phi_f\nu_f + \phi_m\nu_m \\
\frac{\nu_{21}}{E_2} &= \frac{\nu_{12}}{E_1} \\
G_{12} &= \frac{G_f G_m}{G_f\phi_m + G_m\phi_f} \quad \text{Shear Modulus}
\end{aligned}
$$

- ϕ_f is the fibre volume fraction
- $\phi_m = 1 - \phi_f$ is the matrix volume fraction

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3 Revision

3.1 Failure Criteria For Unidirectional FRP

- Brittle Fibres – Ductile Matrix

$$
\begin{array}{lll}
\rightarrow & \sigma_1^* = (1-\phi_f)\sigma_m^* & \text{Low } \phi_f \\
\rightarrow & \sigma_1^* = \phi_f\sigma_f^* + (1-\phi_f)\sigma_m' & \text{High } \phi_f
\end{array}
$$

- Ductile Fibres – Brittle Matrix

$$
\begin{array}{lll}
\rightarrow & \sigma_1^* = \phi_f\sigma_f' + (1-\phi_f)\sigma_m^* & \text{Low } \phi_f \\
\rightarrow & \sigma_1^* = \phi_f\sigma_f^* & \text{High } \phi_f
\end{array}
$$

Note there was a typo in the presentation of the second of these expressions in last week's handout.

σ_f' is stress in fibre at failure strain of matrix, ϵ_m^*
σ_m' is stress in matrix at failure strain of fibre, ϵ_f^*

4 Failure of Unidirectional Composite Laminates

(a) Brittle Fibre, Ductile Matrix, e.g. Carbon fibre reinforced Epoxy.

(b) Brittle Matrix, Ductile Fibre, e.g. Glass fibre reinforced thermoset-polyester

Figure 1: Fibre and Matrix Failure

5 Failure of Unidirectional Composite Laminates

5.1 Brittle Fibres: Failure Criteria

$\sigma_1^* = (1-\phi_f)\sigma_m^*$ Low ϕ_f

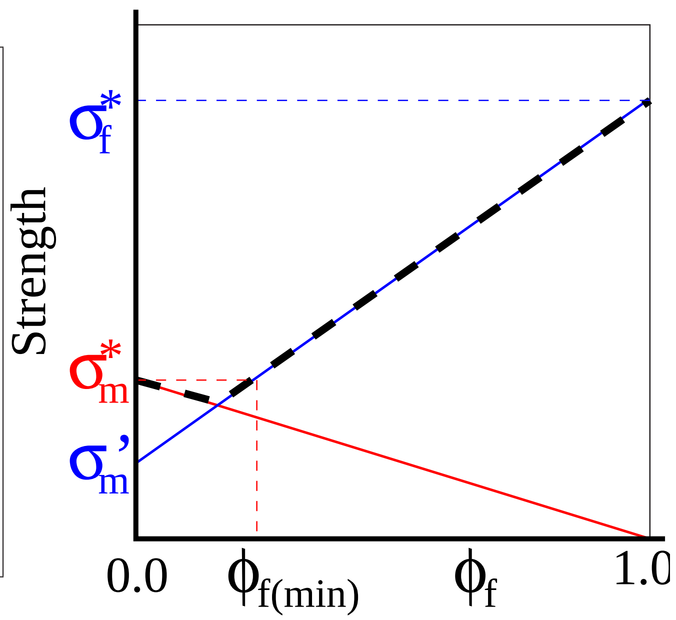
$\sigma_1^* = \phi_f\sigma_f^* + (1-\phi_f)\sigma_m'$ Big ϕ_f

Note: $\sigma_m' = \sigma_m(\epsilon_f^*)$

- Use whichever is higher (heavy dashed line in Figure).
- Note v. small ϕ_f **weakens** matrix! But it will still stiffen the matrix somewhat.

$$\phi_{f_{\min}} = \frac{\sigma_m^* - \sigma_m'}{\sigma_f^* - \sigma_m'}$$

- For carbon-fibre reinforced epoxy $\phi_{f_{\min}} = 0.03$.



6 Failure of Unidirectional Composite Laminates

6.1 Brittle Matrix: Failure Criteria

Follow broadly simiar reasoning to brittle fibre case. Essentially switch roles of fibre and matrix in arguments

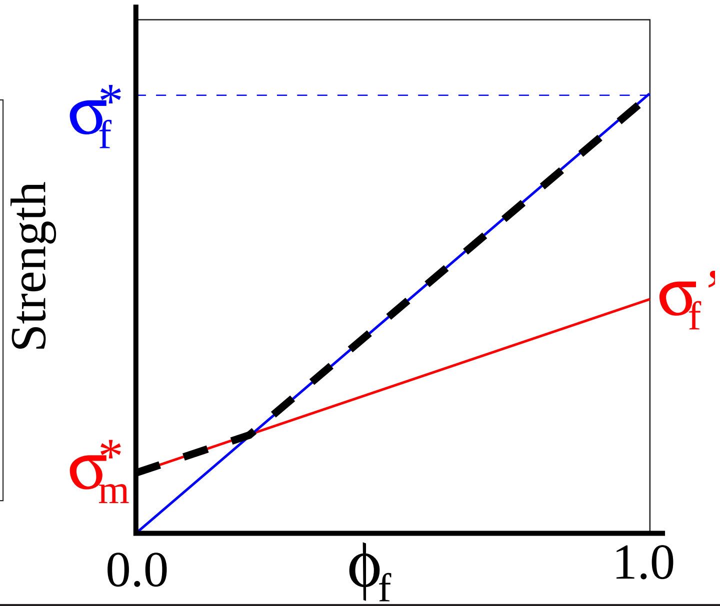
Example: Glass fibre reinforced thermoset polyester

$\sigma_1^* = \phi_f \sigma_f^*$ High ϕ_f

$\sigma_1^* = \phi_f \sigma_f' + (1 - \phi_f)\sigma_m^*$ Low ϕ_f

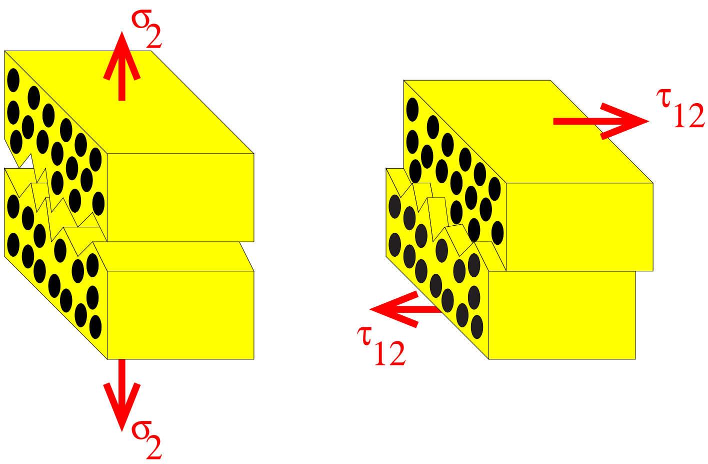
Note: $\sigma_f' = \sigma_f(\epsilon_m^*)$

- Use whichever is higher (heavy dashed line in Figure).
- Note, in this case fibre always strengthens matrix



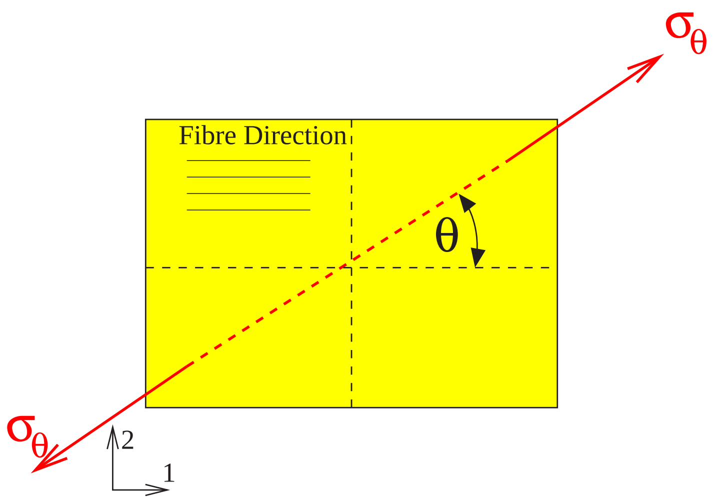
7 Failure of Unidirectional Composite Laminates

When loads are not parallel to the fibres, composite is much weaker. The high strengths of composites are realied only when the loads are parallel to the fibres. The composite is much weaker under stress in other directions becasue cracks seek out the easiest path along which to popragate. In a Fibre Reinforced Polymer, this will be through the matrix and along the the fibre–matrix interface. When a tensile stress acts transversely to the fibres, fracture can occur without fibre fracture.



8 Failure of Unidirectional Composite Laminates

8.1 Loading at angle θ to fibres



In fact, the fibres can act as stress concentrators, so composite is somewhat weaker than matrix alone.
From Mohr's Circle, given σ_θ we have components:

$$\sigma_1 = \sigma_\theta \cos^2\theta$$
$$\sigma_2 = \sigma_\theta \sin^2\theta$$
$$\tau_{12} = \sigma_\theta \sin\theta\cos\theta$$

Failure occurs when any one of these three reaches its limiting value

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9 Failure of Unidirectional Composite Laminates

9.1 Loading at angle θ to fibres

Failure occurs as soon as any of the following occur:

Axial Tensile Failure	$\sigma_\theta^* = \sigma_1^*/\cos^2\theta$
Transverse Tensile Failure	$\sigma_\theta^* = \sigma_2^*/\sin^2\theta$
Axial Shear Failure	$\sigma_\theta^* = \tau_{12}^*/(\sin\theta\cos\theta)$

i.e. we have three possible modes of failure, which gives rise to three failure stresses σ_θ^*. Whichever value of σ_θ^* is the lowest will be the one that leads to failure first. The values of σ_1^*, σ_2^*, τ_2^* are all properties of the unidirectional composite. σ_θ^* is a property of the plate and also of the direction of loading.

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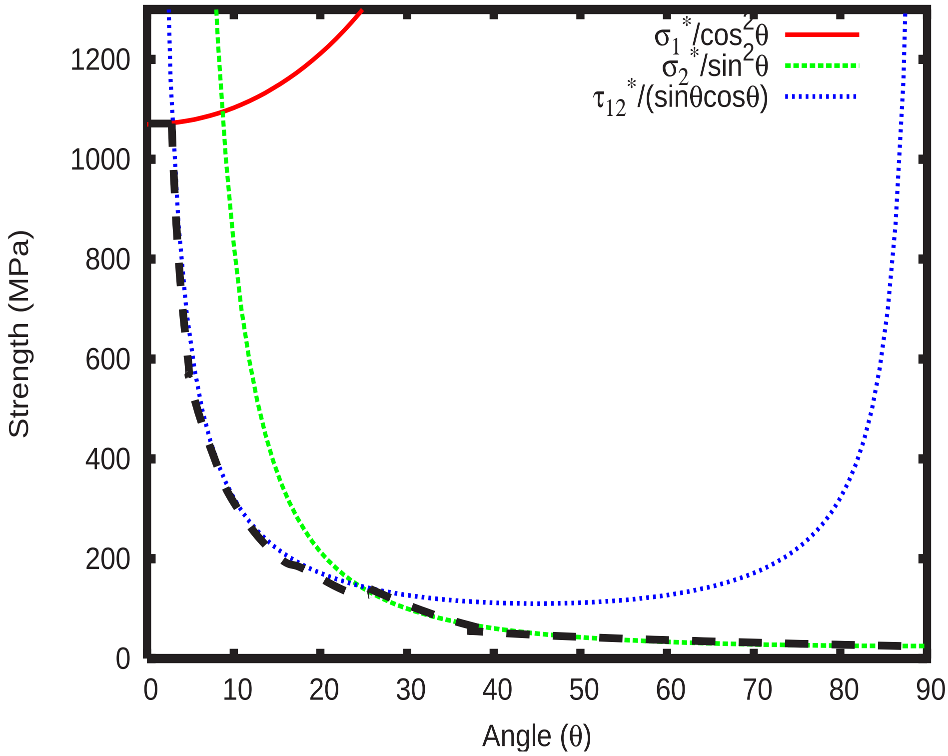
10 Sample Problem

10.1 Example 6/8

A composite material comsists of 55% by volume continuous uniaxially aligned S-glass fibres in a matrix of epoxy. Such a composite is found to have a tensile strength transverse to the fibres $\sigma_2^* = 25$ MPa and shear stength parallel to the fibres of $\tau_{12}^* = 55$ MPa. The tensile strength and modulus of the fibres are 1900 MPa and 86 GPa, and of the matrix are 60 MPa and 2.4 GPa respectively. The composite is to be subjected to tensile stress in a direction inclined at $20°$ to the fibre axes. Predict the stress at failure, and the mode of failure.

11 Failure of Unidirectional Composite Laminates

11.1 Loading at angle θ to fibres



Heavy dashed line is plate strength

Note the huge difference in strength between the fibre direction and other directions in the plate. Results based on this theory agree quite well with experiment, as can be seen in Fig. 6.21 of Principles of Polymer Engineering, McCrum and Buckley.

Not only can we predict the stress at which failure will occur, but we can also anticipate the mode of failure.

Some worked examples followed in class.

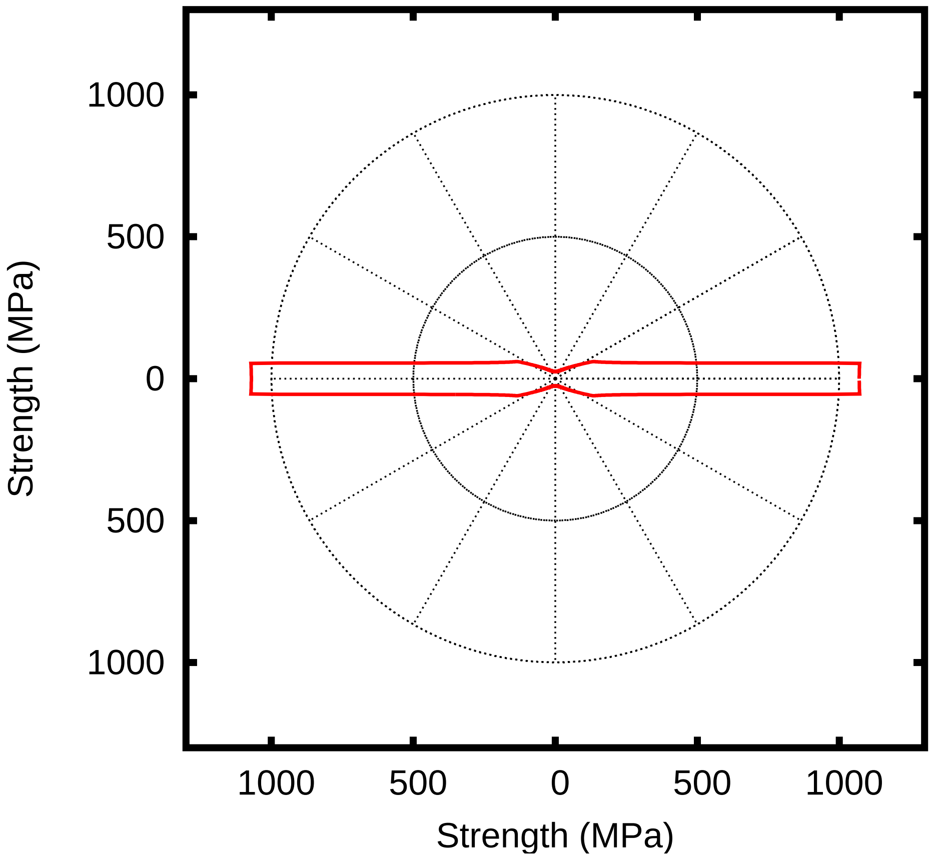


Figure 2: Polar plot of strength as a function of direction for a unidirectional fibre reinforced composite (material properties are as given in example 6.8)