1 3rd Year Engineering Materials

Viscoelasticity – Lecture 4

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2 Viscoelastic Behaviour

2.1 Boltzmann Superposition Principle

- For a Linear Viscoelastic material, this proposes that
 - Strain response due to complex loading is the sum of the strains due to each step

$$\operatorname{Modulus} = E(t) = \frac{\sigma}{\epsilon(t)}$$

If stress σ_0 applied at zero time...

$$\Rightarrow \text{creep strain} = \epsilon(t) = \frac{\sigma_0}{E(t)}$$

If stress σ_1 applied at time $u \dots$

$$\Rightarrow \text{creep strain} = \epsilon(t) = \frac{\sigma_1}{E(t-u)}$$

3 Viscoelastic Behaviour

3.1 Boltzmann Superposition Principle

Suppose there is a series of stress increments, starting at t=0

$$\epsilon(t) = \sum_{i=0}^{N} \frac{\sigma_i}{E(t - u_i)}$$

In limit, this becomes an integration

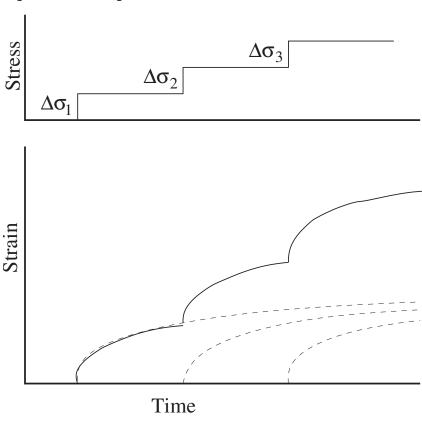
$$\Rightarrow \epsilon(t) = \int_0^N \frac{1}{E(t-u)} \left(\frac{\mathrm{d}\sigma(u)}{\mathrm{d}u} \right) \mathrm{d}u$$

and...

$$\Rightarrow \sigma(t) = \int_0^N E(t - u) \left(\frac{\mathrm{d}\epsilon(u)}{\mathrm{d}u}\right) \mathrm{d}u$$

4 Viscoelastic Behaviour

4.1 Boltzmann Superposition Principle



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