# 1 3rd Year Engineering Materials

# Viscoelasticity – Lecture 4

## 2 Viscoelastic Behaviour

### 2.1 Boltzmann Superposition Principle

- For a Linear Viscoelastic material, this proposes that
  - Strain response due to complex loading is the sum of the strains due to each step

Modulus $= E(t) = \dfrac{\sigma}{\epsilon(t)}$

If stress $\sigma_0$ applied at zero time…

$$\Rightarrow \text{creep strain} = \epsilon(t) = \frac{\sigma_0}{E(t)}$$

If stress $\sigma_1$ applied at time $u$ …

$$\Rightarrow \text{creep strain} = \epsilon(t) = \frac{\sigma_1}{E(t-u)}$$

## 3 Viscoelastic Behaviour

### 3.1 Boltzmann Superposition Principle

Suppose there is a series of stress increments, starting at $t = 0$

$$\epsilon(t) = \sum_{i=0}^{N} \frac{\sigma_i}{E(t-u_i)}$$

In limit, this becomes an integration

$$\Rightarrow \epsilon(t) = \int_0^N \frac{1}{E(t-u)} \left( \frac{\mathrm{d}\sigma(u)}{\mathrm{d}u} \right) \mathrm{d}u$$

and…

$$\Rightarrow \sigma(t) = \int_0^N E(t-u) \left( \frac{\mathrm{d}\epsilon(u)}{\mathrm{d}u} \right) \mathrm{d}u$$

# 4 Viscoelastic Behaviour

## 4.1 Boltzmann Superposition Principle