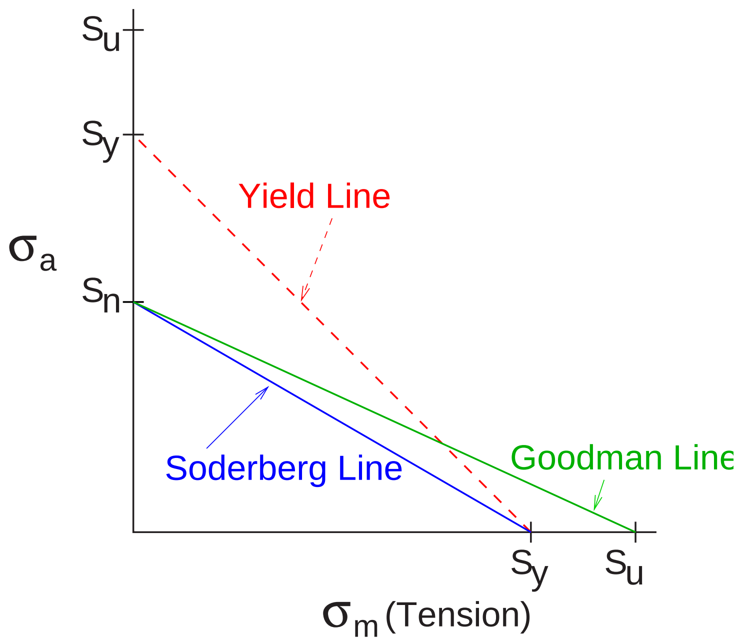
1 3rd Year Design and Production

Fatigue – Lecture 4

2 Constant Life Fatigue Diagrams



3 Constant Life Fatigue Diagrams

3.1 Equations

- Soderberg Line is based on S_y and S_n

  $\rightarrow$ $S_a/S_n + S_m/S_y = 1$

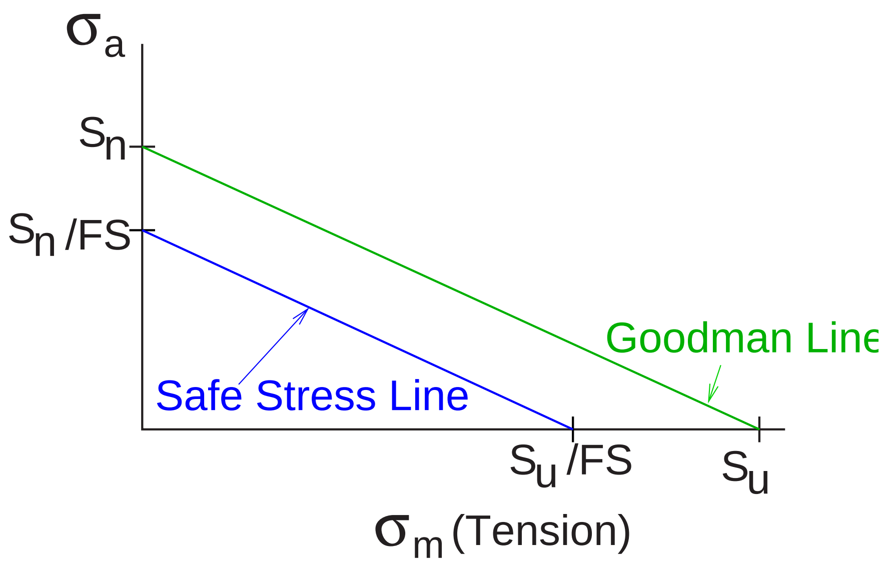
- Goodman Line is based on S_u and S_n

  $\rightarrow$ $S_a/S_n + S_m/S_u = 1$

4 Factor of Safety

- Illustrated here for Goodman Criteria (can use Soderberg, etc., also).
- FS is the factor of safety
- Safe stress line is parallel to original Goodman line

$$\frac{S_a}{S_n} + \frac{S_m}{S_u} = \frac{1}{FS}$$



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5 Summary of CLF Diagrams

- Four straight lines relevant for infinite life:
  - Soderberg, Goodman, Yield-line, Load-line
- Which failure criteria to use?:
  - If no yielding allowed, Soderberg will be conservative
  - If we don't know S_u, then use Soderberg
  - If S_u **is** known, and we want to find infinite life, with yielding permitted, then use Goodman line
  - If S_u **is** known, and we want to find infinite life, with yield onset permitted, then use Goodman line and yield line to define the safe zone (Modified Goodman Criterion).

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6 Stress Concentration – Geometry

- Surface features and flaws can lead to locally elevated stresses
- Tables/charts allow us to look up K_t
  - Theoretical or Geometric stress concentration factors
- Materials have different **notch sensitivities**: q
  - Material with a lot of flaws already are not damaged much by one more
    $\Rightarrow$ low notch sensitivity
  - Very perfect material is significantly damaged by addition of a notch or flaw
    $\Rightarrow$ high notch sensitivity
- Calculate **Fatigue Stress Concentration Factor** K_f using K_t and q:

  $$K_f = 1 + q(K_t - 1)$$

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7 Stress Concentration

Now that we have K_f what do we do with it?

- Ductile Materials
  - Nominal Mean Stress Method
  - Apply K_f only to alternating stress
- Brittle Materials
  - Residual Stress Method
  - Apply K_f to both alternating stress and mean stress

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8 Stress Concentration

8.1 Applied to Goodman Criteria

- Nominal Mean Stress Method (ductile material)

  Apply K_f only to alternating component

  $$\frac{[K_f S_a]}{S_n} + \frac{S_m}{S_u} = \frac{1}{FS}$$

- Residual Stress Method (brittle material; adjust for yielding and resultant residual stress if predicted stress $> S_y$)

  Apply K_f to alternating **and** mean components

  $$\frac{[K_f S_a]}{S_n} + \frac{[K_f S_m]}{S_u} = \frac{1}{FS}$$

Different texts will make different recommendations on this.

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9 Equivalent Stress Equations

To account for situation where there is a combination of bending, shear, and/or axial stresses it is necessary to determine the equivalent stress that is created. Different forms are possible…

- Maximum Shear Stress

$$\tau_{\mathrm{max}} = \sqrt{\left(\frac{\sigma_{\mathrm{eq}}}{2}\right)^2 + \tau_{\mathrm{eq}}^2}$$

- Maximum Normal/Principle Stress

$$\sigma_{\mathrm{max}} = \frac{\sigma_{\mathrm{eq}}}{2} + \sqrt{\left(\frac{\sigma_{\mathrm{eq}}}{2}\right)^2 + \tau_{\mathrm{eq}}^2}$$

- Von-Mises / Distortion Energy Theory

$$\sigma_{\mathrm{max}} = \sqrt{\sigma_{\mathrm{eq}}^2 + 3\tau_{\mathrm{eq}}^2}$$

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10 Equivalent Stress Equations

10.1 How to Use these Relations

Juvinall recommends the following policy:

- Find the **equivalent <u>alternating</u> bending stress** using distortion energy theory as:

$$\sigma_{\mathrm{ea}} = \sqrt{\sigma_a^2 + 3\tau_{\mathrm{a}}^2}$$

- Find the **equivalent <u>mean</u> bending stress** as the maximum principle stress:

$$\sigma_{\mathrm{em}} = \frac{\sigma_m}{2} + \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2}$$

Shigley recommends the application of the distortion energy equation to both alternating and mean stresses.
