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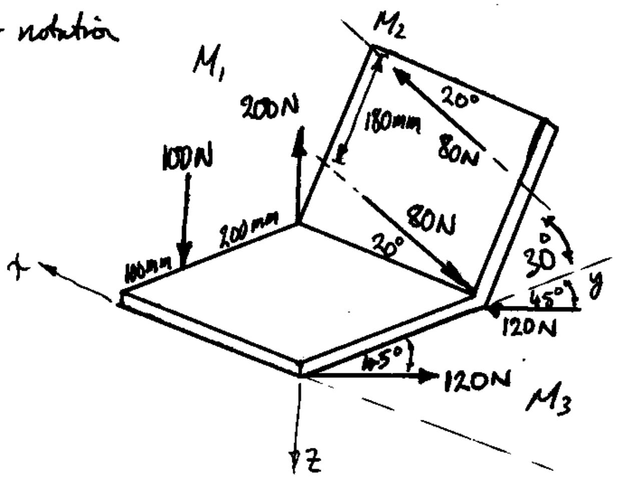
Looking for resultant couple $\vec{M}$

must add 3 couples together ... vector notation

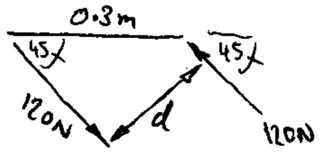


$\|\vec{M}_1\| = (100)(0.2) = 20\ \text{NM}$

$\vec{M}_1 = -20\hat{i}$ ... Right hand rule

(Nm)

now look at $\vec{M}_3$, looking DOWN on top of it



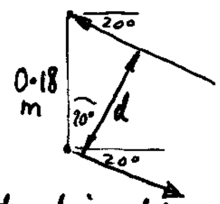
what is d?

$d = 0.3 \cos 45° = 0.212\ \text{m}$

$\therefore \|\vec{M}_3\| = (120)(0.212) = 25.44\ \text{Nm}$

& $\vec{M}_3 = -25.44\hat{k}$ (Nm) again using Right hand rule

finally, $\vec{M}_2$, as before find d



$d = 0.18 \cos 20° = 0.1691\ \text{m}$

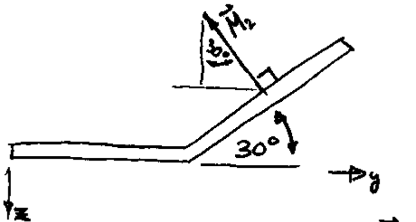
$\therefore \|\vec{M}_2\| = (80)(0.1691) = 13.53\ \text{Nm}$

what about direction? it will be $\perp$ to plane of vectors

$M_{2y} = -\|\vec{M}_2\| \sin(30°) = (13.53)(\tfrac{1}{2}) = -6.77\ \text{Nm}$

$M_{2z} = -\|\vec{M}_2\| \cos(30°) = (13.53)(\sqrt{3}/2) = -11.72\ \text{Nm}$

$\vec{M}_2 = -6.77\hat{j} - 11.72\hat{k}$



we know DIRXN of $\vec{M}_2$ from right hand rule

to find resultant $\vec{M}$, add component of $\vec{M}_1$, $\vec{M}_2$ & $\vec{M}_3$

$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 = -20\hat{i} - 6.77\hat{j} - 11.72\hat{k} - 25.44\hat{k}$

$\underline{= -20\hat{i} - 6.77\hat{j} - 37.16\hat{k}}$

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find resultant force and moment at O

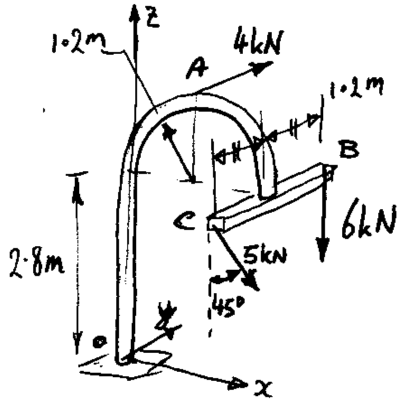
add a few labels : A, B, C

vector notation

$\vec{F}_A = +4\hat{j}$ (kN)

$\vec{F}_B = -6\hat{k}$ (kN)

$\vec{F}_C = \frac{5}{\sqrt{2}}\hat{i} - \frac{5}{\sqrt{2}}\hat{k}$ (kN)



Resultant force $\vec{R}$ is $\Sigma(\vec{F}_A, \vec{F}_B, \vec{F}_C)$

$$\vec{R} = \frac{5}{\sqrt{2}}\hat{i} + 4\hat{j} - (6 + 5/\sqrt{2})\hat{k} \quad \therefore \ \underline{\underline{\|\vec{R}\| = 10.928 \text{ kN}}}$$

now, find moment of each force

$\vec{F}_A$; $\vec{r}_A = 1.2\hat{i} + (2.8+1.2)\hat{k}$ vector from O TO A

$$\vec{M}_A = \vec{r}_A \times \vec{F}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & 0 & 4 \\ 0 & 4 & 0 \end{vmatrix} = -16\hat{i} + 4.8\hat{k}$$

$\vec{F}_B$; $\vec{r}_B = 2.8\hat{k} + 2.4\hat{i} + 1.2\hat{j}$

$$\vec{M}_B = \vec{r}_B \times \vec{F}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.4 & 1.2 & 2.8 \\ 0 & 0 & -6 \end{vmatrix} = -7.2\hat{i} + 14.4\hat{j}$$

$\vec{F}_C$: $\vec{r}_C = 2.4\hat{i} - 1.2\hat{j} + 2.8\hat{k}$

$$\vec{M}_C = \vec{r}_C \times \vec{F}_C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.4 & -1.2 & 2.8 \\ 5/\sqrt{2} & 0 & -5/\sqrt{2} \end{vmatrix} = \frac{6}{\sqrt{2}}\hat{i} + \left(\frac{12}{\sqrt{2}} + \frac{14}{\sqrt{2}}\right)\hat{j} + \frac{6}{\sqrt{2}}\hat{k}$$

$$\therefore \ \vec{M} = \Sigma \vec{M}_A, \vec{M}_B, \vec{M}_C = \hat{i}\left[16 - 7.2 + \frac{6}{\sqrt{2}}\right] + \hat{j}\left[0 + 14.4 + \left(\frac{12}{\sqrt{2}} + \frac{14}{\sqrt{2}}\right)\right] + \hat{k}\left[4.8 + \frac{6}{\sqrt{2}}\right]$$

$$\vec{M} = -18.96\hat{i} + 32.79\hat{j} + 9.04\hat{k}$$

$$\Rightarrow \underline{\underline{\|\vec{M}\| = 38.94}}$$

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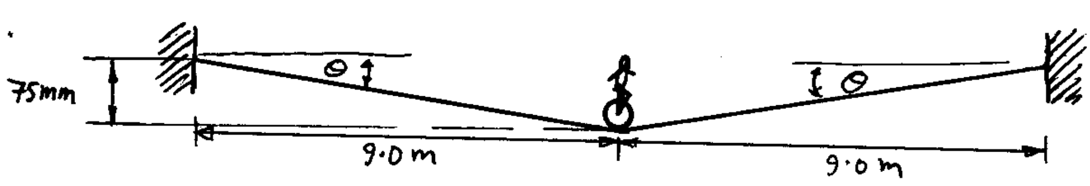
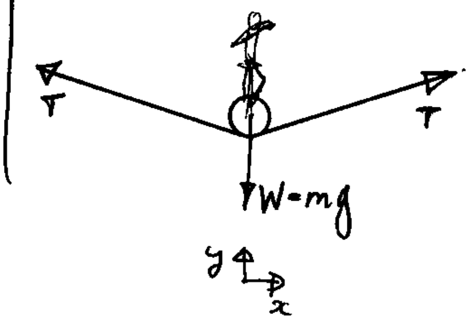


mass of acrobat 50kg

first, what is θ, deflection of tightrope?

$$\theta = \tan^{-1}\left(\frac{0.075}{9.00}\right) = 0.0477^\circ \quad \ldots \text{ v. small angle.}$$



$$W = (50)(9.81) = 490.5\,N$$

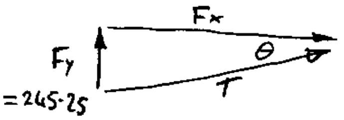
each side of rope bears half of load

$$\frac{490.5\,N}{2} = 245.25\,N$$

this is y component of T

force must be parallel to rope

⇒



$\theta = 0.0477^\circ$

which means that

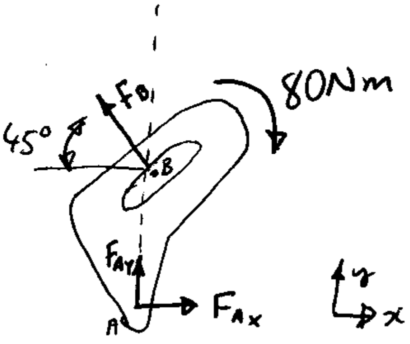
$$\frac{F_x}{F_y} = \frac{9}{0.075}$$

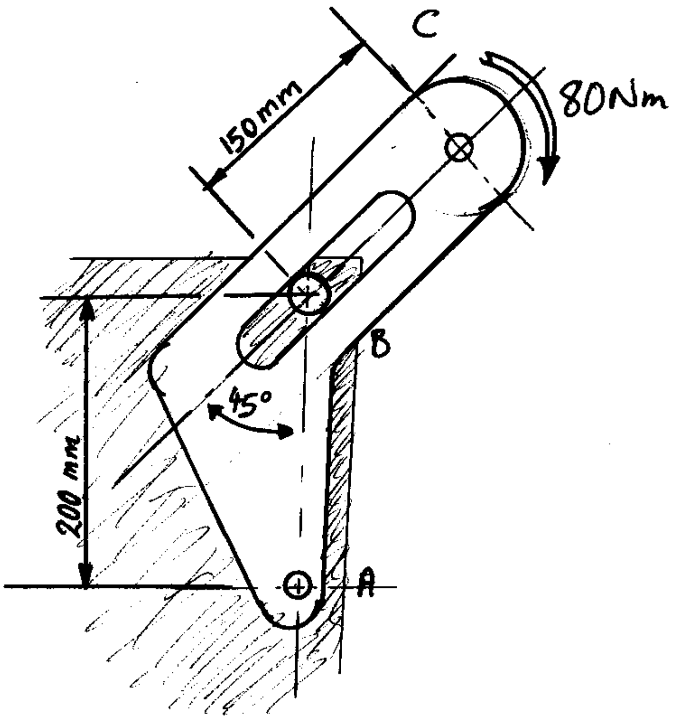
$$F_x = \frac{(9)(F_y)}{0.075} = \frac{(9)(245.25)}{0.075} = 29\,430\,N$$

Tension in rope $T \simeq 29430\,N = 29.43\,kN$

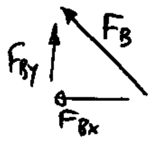
$\boxed{3/96}$ find magnitude of force at pin A.

F.B.D.





$\|F_{Bx}\| = \frac{F_B}{\sqrt{2}}$ $\quad \|F_{By}\| = \frac{F_B}{\sqrt{2}}$



Take moments about A & apply equilibrium

$(F_{Bx})(0.2) - 80\text{ Nm} = 0$ Remember, right hand rule

$\Rightarrow F_{Bx} = 400\text{N}.$

$\Rightarrow F_B = (400)\sqrt{2} = 566\text{ N}$

Take moments about B & apply Equilibrium

$(F_{Ax})(0.2) - 80\text{ Nm} = 0$

$\Rightarrow F_{Ax} = 400\text{ N}$

could also say $\Sigma F_x = 0 \Rightarrow F_{Ax} + F_{Bx} = 0$

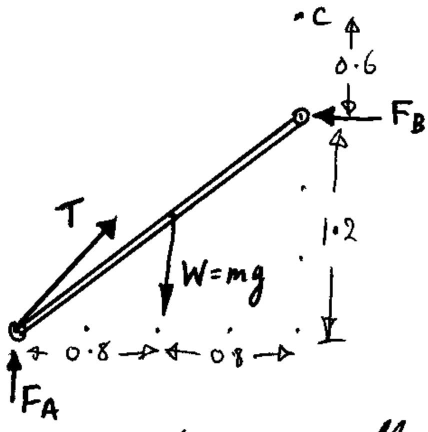
$F_{Ax} = -F_{Bx} = -400$ (i.e. 400 & in opp direction).

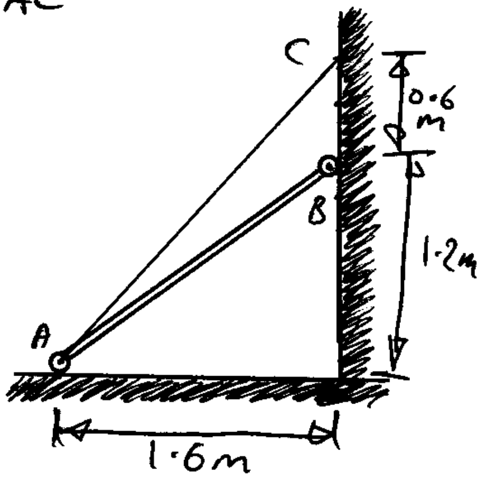
Similarly $F_{Ay} = F_{By}$

$\therefore$ Magn of force @ A = magn of force @ B = $\underline{\underline{566\text{N}}}$

find Rxns @ rollers & tension in rope AC

FBD





assume no friction in rollers.

$$W = (30\,kg)(9.81\,ms^{-2}) = 294.3\,N$$

Take moments about A (because T has no moment about this point).
APPLY EQUILIB $\Sigma M_A = 0$

$$-(W)(0.8) + (F_B)(1.2) = 0$$

$$F_B = \frac{(W)(0.8)}{1.2} = \frac{(294.3)(0.8)}{1.2} = \underline{\underline{196.2\,N}}$$

TAKE MOMENTS ABOUT C... AGAIN T HAS NO MOMENT; APPLY EQUILIB
$\Sigma M_C = 0$

$$(W)(0.8) - (F_B)(0.6) - (F_A)(1.6) = 0$$

$$F_A = \frac{(294.3)(0.8) - (196.2)(0.6)}{1.6} = \underline{\underline{73.575\,N}}$$

NOW Forces

$$\Sigma F_x = 0$$

$$\Rightarrow T_x = F_B = 196.2\,N$$

$$T_y = W - F_A = 294.3 - 73.575 = 220.725$$

$$\|\vec{T}\| = \sqrt{(196.2)^2 + (220.725)^2} = \underline{\underline{295.3\,N}}$$

<u>CHECK</u> SINCE $\vec{T}$ MUST BE PARALLEL TO $\vec{AC}$

We SHOULD HAVE $\frac{T_x}{T_y} = \frac{1.6}{1.8} = 0.8889$

$$\frac{T_x}{T_y} = \frac{196.2}{220.725} = 0.8889 \checkmark$$

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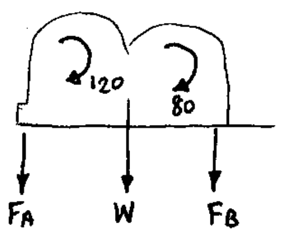
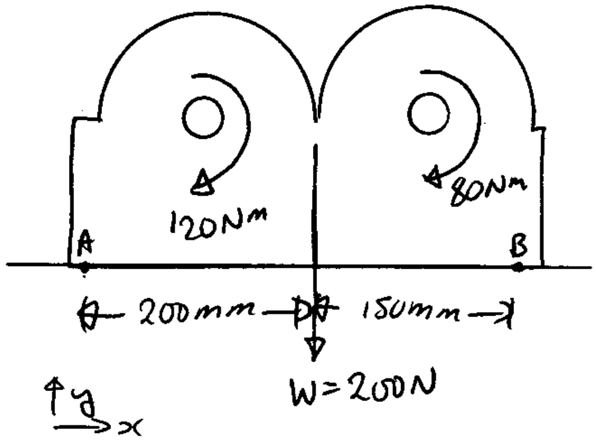
F.B.D.





moment about A & Equilib

$$\sum M_A = 0$$

$$-(0.35)F_B - (0.2)(W) - 120 - 80 = 0$$

$$-F_B = \frac{200 + (0.2)(200)}{0.35} = 685.7\ N$$

$$\underline{\underline{F_B = -682.7\ N}}$$

NOTE SIGN, this means DIRXN is opposite to SHOWN in FBD, i.e. it's UP ↑

$$\sum F_y = 0$$

$$F_A + W + F_B = 0$$

$$F_A = -W - F_B$$

$$F_A = -200 - (-682.7\ N)$$

$$F_A = 682.7 - 200$$

$$\underline{\underline{F_A = +485.7\ N}}$$

SO THIS Force is INDEED DOWNWARDS, as SHOWN.

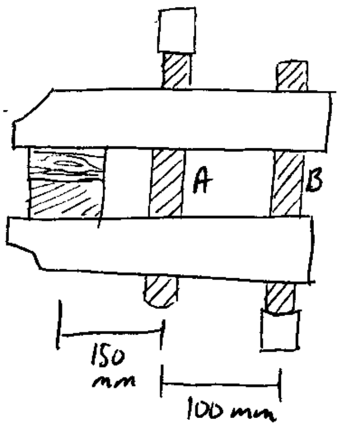
$\boxed{3/19}$

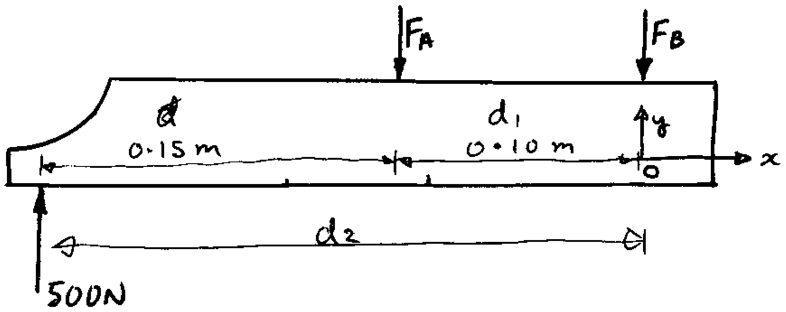
500N compression force on wood

WHAT FORCE IS IN SCREW @ A

CONSIDER HALF OF CLAMP,

and draw free body diagram





note where point "O" has been selected

take moments about O and apply equilibrium condition

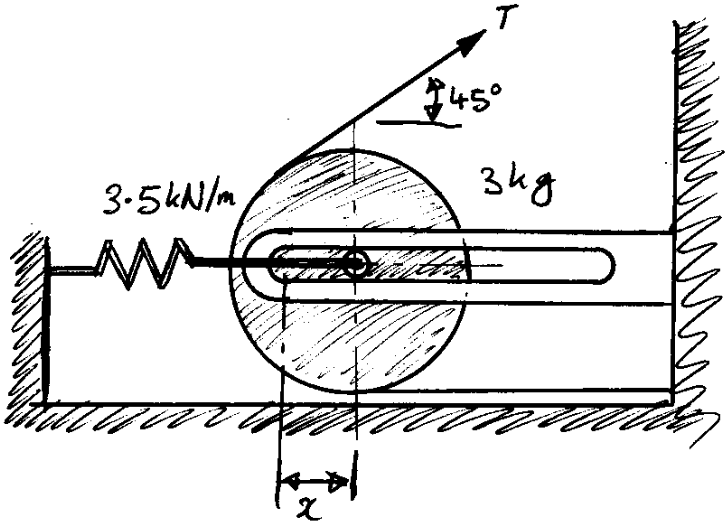
$\sum M_O = 0$

$\Rightarrow +F_A d_1 - (500)(d_2) = 0$

$F_A = (500)\dfrac{d_2}{d_1} = 500\,\dfrac{0.25}{0.10} = \underline{\underline{1250\,\text{N}}}$

3/23

find T if x, the extension in the spring, is 160mm also, what force N is exerted on the slot?

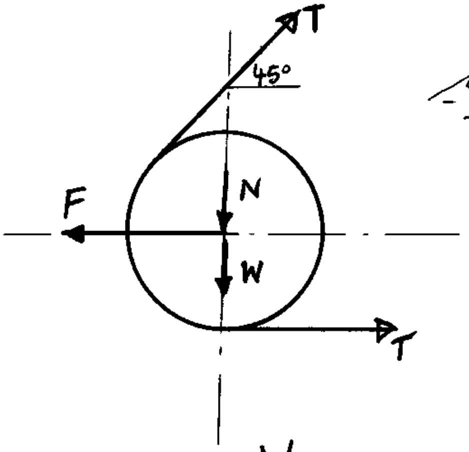


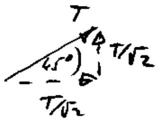
first, Free Body Diagram

assumptions.

pulley smooth ⇒ tension in rope is constant along length

slot smooth ⇒ NO HORIZ FORCE exerted BY slot





Apply Equilib CONDITIONS

$\sum F_x = 0 \Rightarrow T + T/\sqrt{2} = F$

WHAT is F?

$F = (160)(3.5 \times 10^3)/1000$

$F = 560N$

$\Rightarrow T = \dfrac{F}{1 + \frac{1}{\sqrt{2}}} = \underline{\underline{328N}}$

$\sum F_y = 0 \Rightarrow N + W - T/\sqrt{2} = 0$

$N = T/\sqrt{2} - W = \dfrac{328}{\sqrt{2}} - (9.81)(3.0)$

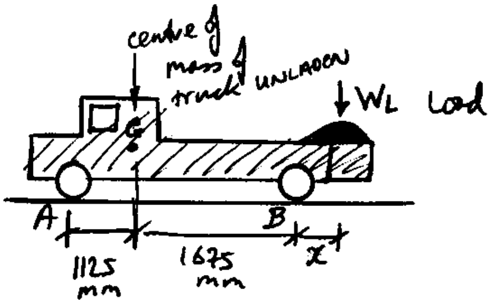
$\underline{\underline{N = 202.5\,N}}$

WHAT ABOUT DIRXN? Well force on pulley is DOWN, so force on SLOT is UPWARDS.

3/23

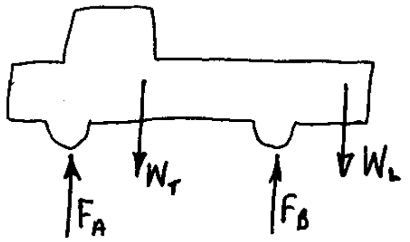
find mass of load such that wheels support equal weight

mass of truck : 1600kg



$x = 400$ mm

F.B.D.



Equilibrium condition

$\sum F_y = 0 \qquad F_A + F_B = W_T + W_L$ we set $F_A = F_B$ so

$$F_B = F_A = \frac{W_T + W_L}{2} \qquad (1)$$

$\sum M_A = 0$

$$F_B\,(1.125 + 1.675) - W_T(1.125) - W_L\,(1.125 + 1.675 + 0.400) = 0 \qquad (2)$$

N.B. RIGHT HAND RULE

Substitute from (1) into (2) for F_B

$$\left(\frac{W_T + W_L}{2}\right)(2.8) - W_T\,(1.125) - W_L\,(3.20) = 0$$

divide across by $g = 9.81 \Rightarrow \frac{W_L}{g} = m_L \quad \frac{W_T}{g} = m_T$

Rearrange too

$$W_L = \frac{W_T\,(1.4 - 1.125)}{1.8} = 1600\,\frac{(0.275)}{(1.800)} = \underline{\underline{244\text{kg}}}$$