Notes: The main equation you need for these problems is in Lecture F, slide 6

you can use

sign depends on right hand rule.

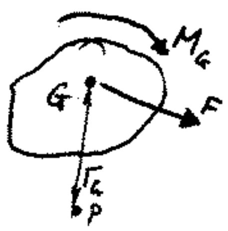
$$\left.\begin{array}{l} ① \ \sum M_P = I_G \alpha \pm m a_G d \\ \text{OR} \\ ② \ \sum M_P = I_P \alpha \pm m a_P d \end{array}\right\} \text{result is same}$$

you know $\vec{a}_P$ ... given in problem

$\vec{a}_G$ is $= \vec{a}_P + \vec{a}_{G/P}$ i.e. accln of P plus relative accln of G rel to P.

① is used in the two worked examples included here.

the principle here is that we look @ effect of all forces about G, & represent it as a Force + a Couple. then for another point P $\sum M_P = \sum M_G + \vec{r}_G \times \vec{F}$



this is a principle we used a lot in the statics part of the course.

Note how we get result of form

$\alpha = (\text{constant})(g \sin\theta + a_0 \cos\theta)$

i.e. when $\theta \approx 0$, α is due mostly to a_0
when $\theta \approx 90$ α is due to g. makes physical sense.

Integration to integrate across a range of θ you need to use

$$\int \omega \, d\omega = \int \alpha \, d\theta$$

$$\Rightarrow \frac{1}{2}\omega^2 = \int \alpha \, d\theta$$

(assumes $\omega = 0$ @ start.)

[rationale: $\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$

$\frac{1}{dt} = \frac{\omega}{d\theta} = \frac{\alpha}{d\omega}$

$\alpha \, d\theta = \omega \, d\omega$]

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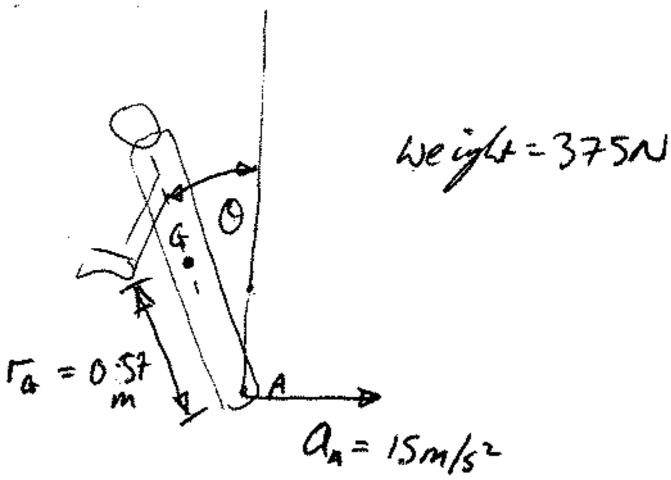
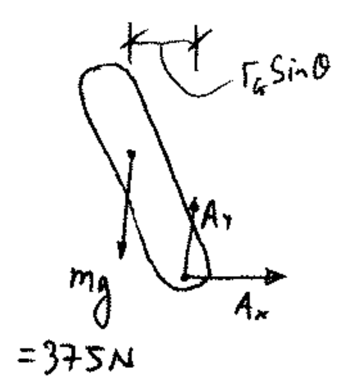
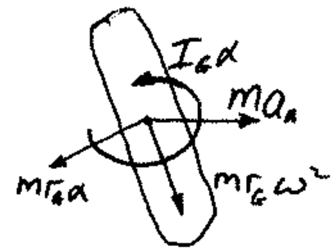
CRASH Dummy
Car Decelerates @ $15\,m\,s^{-2}$
Dummy starts vertical, once
θ is $30°$ find ω of Dummy
Radius of Gyration is 0.21 (k_G)



DRAW F.B.D.



look @ Acceleration at G



for general plane motion, if accel of A known... useful to use the expression

$$\Sigma M_A = [I_G\alpha] + \Sigma m a_G d$$

Note, we use $\Sigma m a_G d$ because $\vec{a}_G$ is made up of $\vec{a}_A$ plus the relative acceleration of G rel to A.

$$\Rightarrow \quad mg\,r_G(\sin\theta) = [mk_G^2\alpha] + [m a_A r_G(\cos\theta) + (mr_G\omega^2)(0) + (mr_G\alpha)(r_G)]$$

cancel out some terms

$$\Rightarrow \quad g\sin\theta = \frac{k_G^2}{r_G}\alpha + a_A\cos\theta + \alpha r_G$$

Rearrange to get $\alpha(\theta) \Rightarrow \alpha = \dfrac{(g\sin\theta + a_A\cos\theta)}{\left(r_G + \frac{k_G^2}{r_G}\right)} = \dfrac{(g\sin\theta + a_A\cos\theta)\,r_G}{r_G^2 + k_G^2}$

need to integrate ... $\theta = 0 \to \theta = 30°$

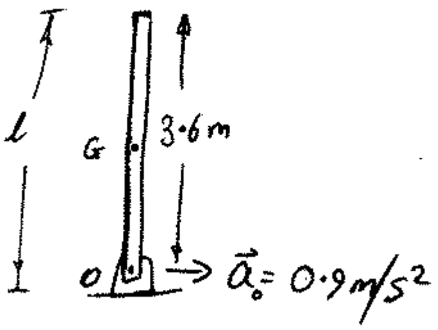
[note $\alpha = \frac{d\omega}{dt}$ $\quad \omega = \frac{d\theta}{dt} \Rightarrow \frac{1}{dt} = \frac{\omega}{d\theta} = \frac{\alpha}{d\omega} \Rightarrow \omega\,d\omega = \alpha\,d\theta$]

$$\int_{\omega_1=0}^{\omega_2=?}\omega\,d\omega = \int_0^{30°}\alpha\,d\theta = \left(\frac{r_G}{r_G^2+k_G^2}\right)\int_0^{30°}(g\sin\theta + a_A\cos\theta)\,d\theta$$

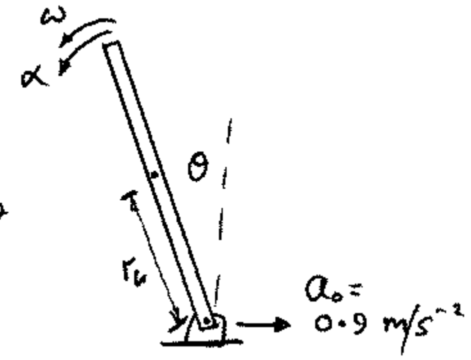
$$\frac{1}{2}\omega^2 = \frac{0.57}{(0.57)^2+(0.21)^2}\left[-g\cos\theta + 15\sin\theta\right]_0^{30} = (1.545)\left(-\frac{(9.81)\sqrt{3}}{2} + \left(\frac{15}{2}\right) + 9.81 - 0\right)$$

$$\frac{1}{2}\omega^2 = 12.85 \Rightarrow \boxed{\omega = \sqrt{(2)(12.85)} = 5.07\ \text{rad}\,s^{-1}}$$

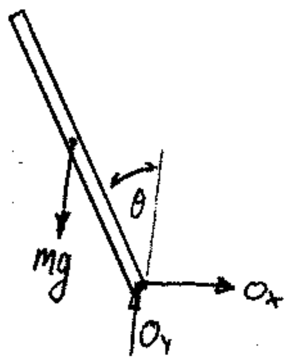
find ω of pole @ Hz posn.



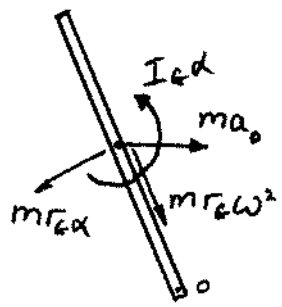
@ θ



F.B. Diagram Forces



(mass)(Accelerations)



acceleration of centre of gravity is sum of acceleration of "O" plus the relative accln of G relative to O (i.e. $mr_G\omega^2$ and $mr_G\alpha$)

$r_G = l/2$

We can use eqn

$$\Sigma M_o = I_G\alpha + \Sigma m a_G d$$

[(a sum because of different components as shown on Figure)]

$$\Sigma M_o = \left(\tfrac{1}{12}ml^2\right)(\alpha) + (ma_o)\left(\tfrac{l}{2}\right)(\cos\theta) + \cancel{(mr_G\omega^2)(0)}^{\,0} + m\left(\tfrac{l}{2}\right)\alpha\left(\tfrac{l}{2}\right)$$

ΣM_o ... calculate moments ... only mg contributes

$$\Sigma M_o = mg\left(\tfrac{l}{2}\right)(\sin\theta)$$

$$\therefore\ \cancel{m}g\left(\tfrac{1\cancel{l}}{\cancel{2}}\right)\sin\theta = {}_6\tfrac{1}{\cancel{12}}\cancel{m}l^{\cancel{2}}\alpha + (\cancel{m}a_o)\left(\tfrac{1\cancel{l}}{\cancel{2}}\right)\cos\theta + \cancel{m}\left(\tfrac{l}{2}\right)\alpha\left(\tfrac{1\cancel{l}}{\cancel{2}}\right)$$

cancel some stuff

$$g(\sin\theta) = \frac{l\alpha}{6} + a_o(\cos\theta) + \frac{l\alpha}{2}$$

$$\therefore\ \alpha\left(\frac{l}{6} + \frac{l}{2}\right) = (g\sin\theta + a_o\cos\theta) = \frac{2l}{3}\alpha = \frac{7{\cdot}2}{3}\alpha = 2{\cdot}4\alpha$$

$$\alpha = \frac{1}{2{\cdot}4}\left(9.81\sin\theta + 0.9\cos\theta\right)$$

cont'd →

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$$\alpha = \frac{1}{2\cdot4}\left(9{\cdot}81 \sin\theta + 0{\cdot}9 \cos\theta\right)$$

need to integrate to get ω. for this we need a relation

$$\int \alpha \, d\theta = \int \omega \, d\omega$$

NOTE $\alpha = \frac{d\omega}{dt}$ $\omega = \frac{d\theta}{dt}$ $\Rightarrow \frac{1}{dt} = \frac{\alpha}{d\omega} = \frac{\omega}{d\theta}$

so $\alpha \, d\theta = \omega \, d\omega$

the same applies for linear accln

$\int a \, ds = \int v \, dv$ ... integrate for const accln

$as = \left[\frac{1}{2}v^2\right]_{v_1}^{v_2} \Rightarrow \boxed{v^2 = u^2 + 2as}$ WELL KNOWN

$$\therefore \int_0^{\omega_2} \omega \, d\omega = \frac{1}{2\cdot4}\int_0^{\frac{\pi}{2}} (9{\cdot}81 \sin\theta + 0{\cdot}9 \cos\theta) \, d\theta$$

$$\frac{1}{2}\omega^2 = \frac{1}{2\cdot4}\left[-9{\cdot}81 \cos\theta + 0{\cdot}9 \sin\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2\cdot4}\left(0{\cdot}9 + 9{\cdot}81\right)$$

$$\omega = \sqrt{\frac{2}{2\cdot4}\left(0{\cdot}9 + 9{\cdot}81\right)} = 2{\cdot}99$$