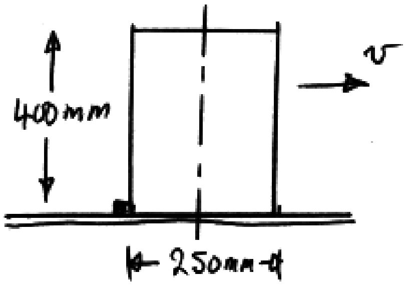
6/17

Look in from side at cylinder on conveyor belt

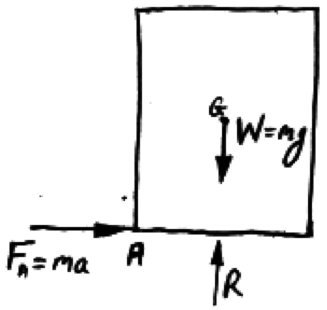


at what value of t will it begin to tip over.

~~at=~~

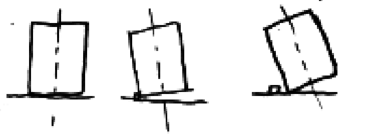
$v = \frac{1}{2} + 0.9t^2$ m/s $\Rightarrow a = \frac{dv}{dt} = 2(0.9)t = 1.8t$

draw F.B.D.



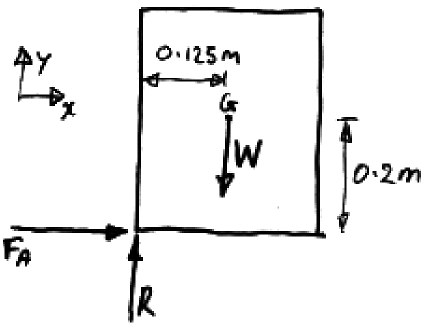
as F_A increases, eventually cylinder will start to tip over point A

i.e.



note, the force $\vec{R}$ is, IN GENERAL, distributed over the bottom of the cylinder, but once it starts to tip, the bottom surface lifts clear and all of R occurs at A

at tipping point F.B.D.



Now, solve...

$\Sigma F_y = 0 \Rightarrow W - R = 0$

$\Rightarrow R = +W = +mg$ (IN DIRECTION DRAWN ON F.B.D.)

$\Sigma M_G = 0$

$\Rightarrow (F_A)(0.2) = (R)(0.125)$

$F_A = \cancel{m}a = \frac{(\cancel{m}g)(0.125)}{0.2} = 6.13m$

$\Rightarrow a = 6.13\ ms^{-2}$

Recall $a = 1.8t = 6.13\ ms^{-2} \Rightarrow \boxed{t = \frac{6.13}{1.8} = 3.41\ s}$

6/20 find vertical force @ TOWBAR.

we are told car and trailer reach 60km/h in 30metres

$60\ km/h = \frac{(60)(1000)}{(60)(60)}\ m/s = 16.667\ m/s$; assume accln constant

recall $v^2 = u^2 + 2as$

$v = 16.667\ m/s$
$u = 0$ (start from dead stop)
$s = 30\ m$
$a = ?$

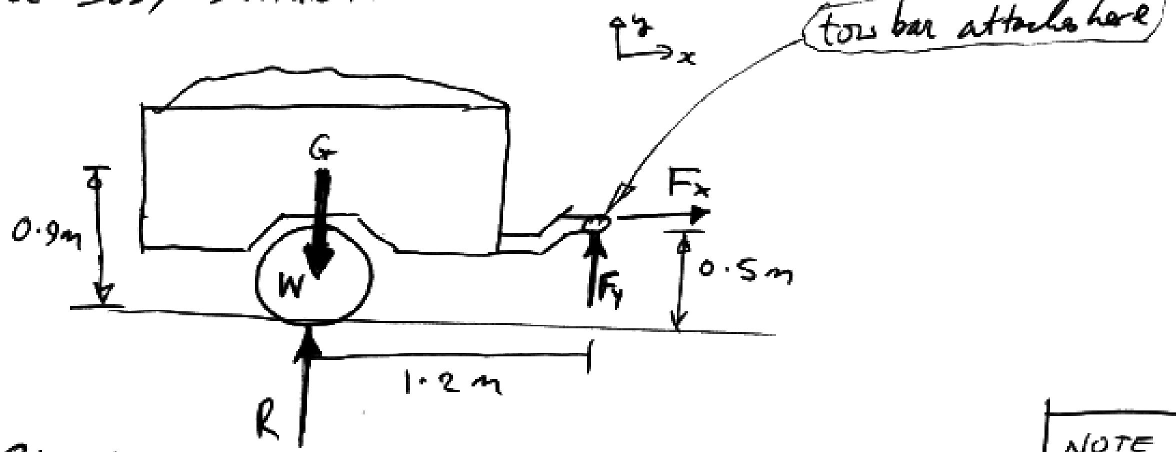
so $a = \frac{v^2}{2s}$
$= \frac{(16.667)^2}{(2)(30)}$

$a = 4.63\ m/s^2$

then $F = ma$, $m = 900kg$

$F = (900)(4.63) = \underline{4166.67\ N}$

Free Body Diagram



Clearly the trailer is in translation

NOTE how $\vec{W}$ & $\vec{R}$ have no moment about G

$\Rightarrow \sum M_G = 0$ so $(F_y)(1.2) - (F_x)(0.9 - 0.5) = 0$

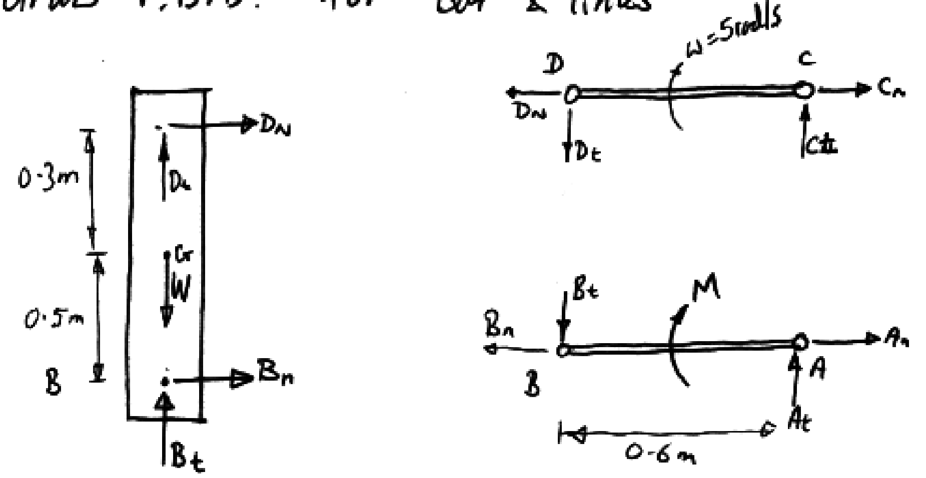
$F_y = \frac{(F_x)(0.4)}{1.2} = \frac{F_x}{3}$ $F_x = 4166.67N$

$$\boxed{F_y = \frac{4166.67}{3} = 1388.9\ N}$$

6/26 Find α for Links & Force acting at D

as soon as you see a parallelogram linkage like this you should think "translation" $\Rightarrow \Sigma M_G = 0$

draw F.B.D. for Bar & links



$W = mg = (25)(9.81)$
$= 245.25N$

Look at [DC], normally we say $\Sigma F = ma$ & $\Sigma M = I\alpha$, but this link is LIGHT so its mass $= 0 \Rightarrow \Sigma F = 0$ & $\Sigma M = 0$

$\Rightarrow$ $D_t = C_t = 0$ & $D_n = C_n$

N.B.

for [BA], similar story

$\Sigma M_A = 0 \Rightarrow (B_t)(0.6) = M = 200Nm$

$B_t = \frac{200}{0.6} = 333.33 N$

at this point we can find a_t for the bar

$\Sigma F_t = ma_t$ or $a_t = \frac{\Sigma F_t}{m}$

$a_t = \frac{B_t + D_t - W}{m} = \frac{333.33 + 0 - 245.25}{25} = 3.52 m/s^2$

$\boxed{\alpha = \frac{a_t}{r} = \frac{3.52}{0.6} = 5.87 \, rad \, s^{-2}}$

finally, we need to get D_n,

Since $\Sigma M_G = 0$ (TRANSLATION) $\Rightarrow (B_n)(0.5) = (D_n)(0.3)$

$B_n = D_n\left(\frac{0.3}{0.5}\right)$

also $\Sigma F_n = ma_n = mr\omega^2$

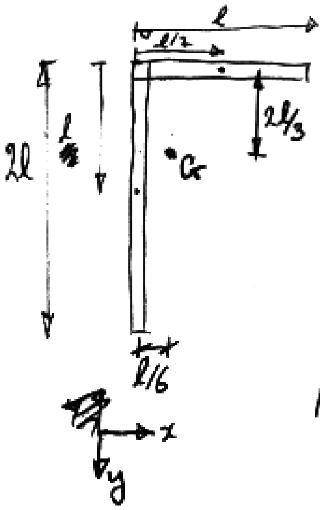
$B_n + D_n = mr\omega^2 \Rightarrow D_n = \frac{mr\omega^2}{\left(1 + \frac{0.3}{0.5}\right)} = \frac{(25)(0.6)(5^2)}{\left(\frac{0.8}{0.5}\right)}$

$\boxed{D_n = 234 N}$ (Since $D_t = 0$ force @ $D \equiv D_n$)

6.27 first, find centre of mass of the L shaped Bar.

if it has total mass m, then the short section has mass $\left[\frac{m}{3}\right]$

and the long section has mass of $\left[2\,m/3\right]$

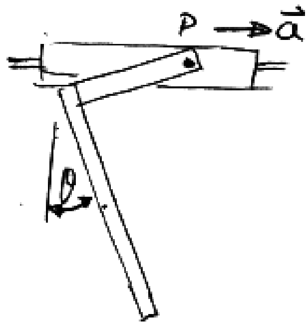


C. of mass G, G_x, G_y

$$G_x = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{(m/3)(l/2) + (2m/3)(0)}{m} = \frac{l}{6}$$

$$G_y = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{(m/3)(0) + (2m/3)(l)}{m} = \frac{2l}{3}$$

It is important to sketch the solution & see if it <u>Looks right</u>

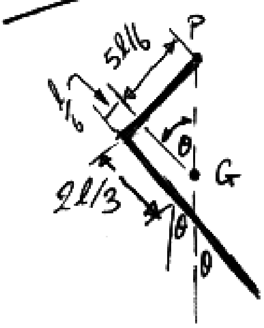


"L" is attached @ P to a slider & the assembly has a constant accln of $\vec{a}$ as shown. we are asked for <u>steady state</u> values of θ

$\Rightarrow \theta$ not changing $\Rightarrow \omega = 0,\ \alpha = 0 \Rightarrow$ TRANSLATION

so $\Sigma M_G = 0$

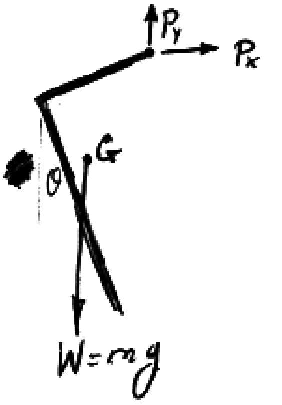
<u>first</u> case $\vec{a} = \vec{0} \Rightarrow$ centre of mass hangs directly below P



note how $\tan(\theta) = \dfrac{5l/6}{2l/3} = \dfrac{5}{6}\dfrac{3}{2} = \dfrac{5}{4} = 1.25$

$\Rightarrow \boxed{\theta = 51.3°}$

Second case $\vec{a} = g/2$ Sketch Free Body Diagram

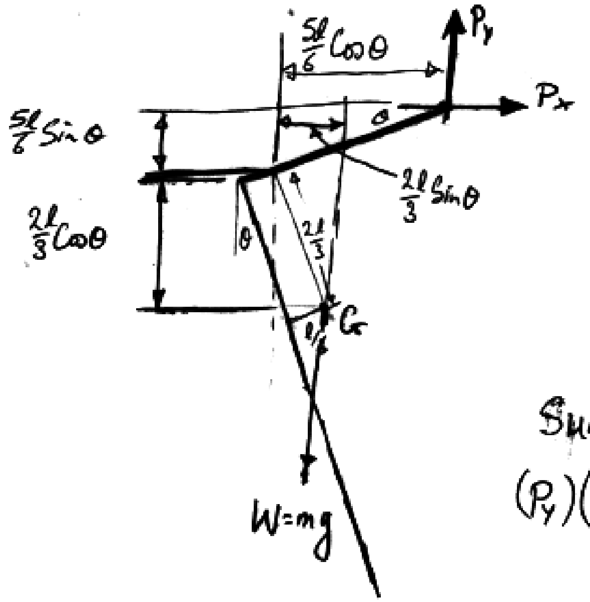


$\Sigma F_y = 0 \Rightarrow P_y = W = mg$

$\Sigma M_G = 0$

↳ tricky thing here is calculating the moment arms as a fn of θ

<u>see next page</u>



So moment arm of P_x about G is $\left[\frac{5\ell}{6}\sin\theta + \frac{2\ell}{3}\cos\theta\right]$

and moment arm of P_y about G is $\left[\frac{5\ell}{6}\cos\theta - \frac{2\ell}{3}\sin\theta\right]$

Sum moments about G $\Sigma M_G = 0$

$$(P_y)\left(\frac{5\ell}{6}\cos\theta - \frac{2\ell}{3}\sin\theta\right) = (P_x)\left(\frac{5\ell}{6}\sin\theta + \frac{2\ell}{3}\cos\theta\right) \circledast$$

now, $\Sigma F_x = ma_x = \frac{mg}{2} \Rightarrow P_x = \frac{mg}{2}$

$\Sigma F_y = ma_y = 0 \Rightarrow P_y - W = 0$

$P_y = W = mg$ substitute these into $\circledast$

$$(mg)\left(\frac{5\ell}{6}\cos\theta - \frac{2\ell}{3}\sin\theta\right) = \frac{mg}{2}\left(\frac{5\ell}{6}\sin\theta + \frac{2\ell}{3}\cos\theta\right)$$

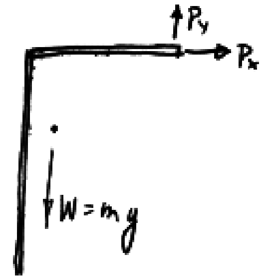
cancel out m, ℓ and g

$$\frac{5}{6}\cos\theta - \frac{2}{3}\sin\theta = \frac{5}{12}\sin\theta + \frac{3}{8}\cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{5/6 - 2/6}{5/12 + 2/3}\right) = 0.462 \Rightarrow \boxed{\theta = 24.78^\circ}$$

Third Case is simpler... find $\|\vec{a}\|$ for $\theta = 0$

$\theta = 0 \Rightarrow$



again $P_y = mg$
$P_x = ma$

$\Sigma M_G = 0 \Rightarrow$

$(P_y)(5/6)(\cancel{\ell}) - (P_x)(2/3)(\cancel{\ell}) = 0$ cancel out "ℓ"

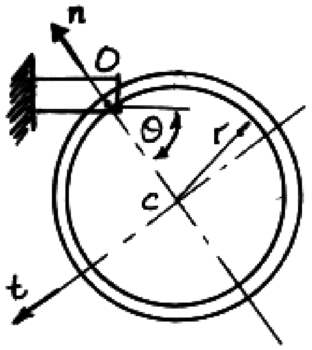
$\Rightarrow \cancel{m}g\frac{5}{6} - \cancel{m}a\frac{2}{3} = 0$ cancel out "m"

$a = g\frac{5}{6}\frac{3}{2} = 5/4\, g$

$$\boxed{a = \frac{5}{4}g \text{ for } \theta = 0}$$

6/55 Narrow ring of mass m is free to rotate about O

find n and t components of force at O as a function of θ (ring is released at $\theta = 0$)

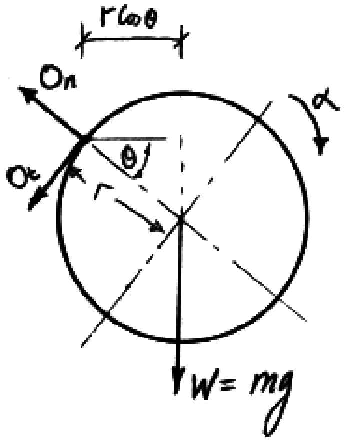


$I_c = I_G = mr^2$ (all mass is at radius r)

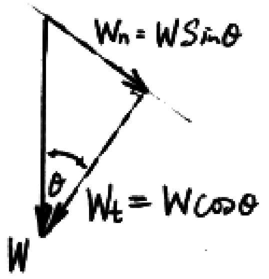
$I_O = I_G + mr^2 = 2mr^2$

NB. note that we are given directions for n and t. Be careful with signs later!

Draw Free Body Diagram for angle θ



look at $\vec{W}$



fixed axis Rotation $\Rightarrow$ $\sum M_O = I_O \alpha$

$mgr \cos\theta = 2mr^2\alpha \Rightarrow \boxed{\alpha = \frac{g\cos\theta}{2r}}$

find ω as a fn of α... Note $\frac{d\omega}{dt} = \alpha$; $\frac{d\theta}{dt} = \omega$ OR $dt = \frac{d\omega}{\alpha}$ and $dt = \frac{d\theta}{\omega}$

equate.... so $\frac{d\omega}{\alpha} = \frac{d\theta}{\omega} \Rightarrow \omega\, d\omega = \alpha\, d\theta$

$\therefore \int \omega\, d\omega = \frac{1}{2}\omega^2 = \int_0^\theta \alpha\, d\theta = \int \frac{g\cos\theta}{2r}\, d\theta = \frac{g\sin\theta}{2r}$

$\boxed{\omega = \sqrt{\frac{g\sin\theta}{r}}}$ ← Note... Energy considerations would allow us to derive this too

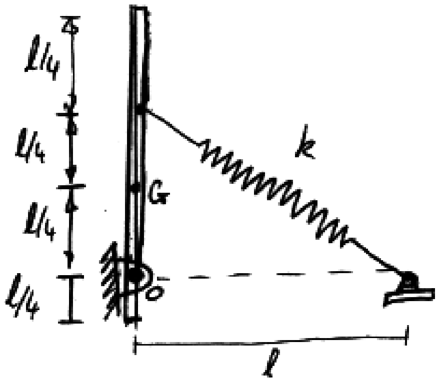
for forces...

$\sum M_G = I_G\alpha$ $\quad -(O_t)(r) = mr^2\frac{g\cos\theta}{2r} \Rightarrow \boxed{O_t = \frac{-mg\cos\theta}{2}}$

$\sum F_n = ma_{G_N}$ $\quad O_n - W_n = mr\omega^2$

$\Rightarrow O_n = mr\left(\frac{g\sin\theta}{r}\right) + mg\sin\theta \Rightarrow \boxed{O_n = 2mg\sin\theta}$

6/56



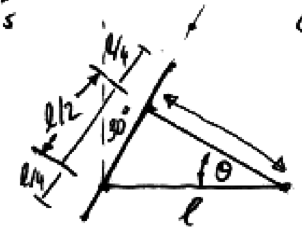
as shown, spring is relaxed (neither compressed nor stretched).

if bar is rotated 30° clockwise, & released, find initial value for α

[assume spring is LIGHT & doesn't sag]

to find force in spring need its new length...

sketch is useful



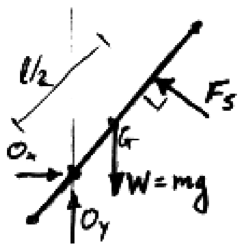
clearly $\theta = 30°$ & length is $l\frac{\sqrt{3}}{2}$

initial length?

$\hookrightarrow \sqrt{l^2 + \left(\frac{l}{2}\right)^2} = l\frac{\sqrt{5}}{2}$

change in length is $l\left[\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right]$ & force is $kl\left[\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right] = F_s$

now ... F.B.D. @ point of release



fixed axis rotation so

$$\sum M_o = I_o \alpha$$

O_x & O_y have no moment about O

moment of F_s is $\left(\frac{l}{2}\right)\left(kl\left(\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right)\right)$ counter clockwise

W is $\left(\frac{l}{4}\right)(\sin(30))mg$ clockwise

what is I_o?

well $I_G = \frac{1}{12}ml^2$; use parallel axis theorem to get I_o

$$I_o = I_G + md^2 = \frac{1}{12}ml^2 + m\left(\frac{l}{4}\right)^2 = \frac{1}{12}ml^2 + \frac{ml^2}{16} = \frac{7}{48}ml^2$$

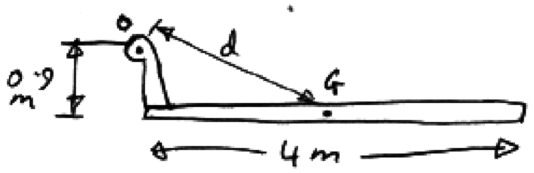
$$\sum M_o = I_o\alpha \Leftrightarrow \alpha = \frac{\sum M_o}{I_o} = \frac{\frac{l}{2}kl\left(\frac{\sqrt{5}}{2} - \frac{\sqrt{3}}{2}\right) - \frac{l}{4}\frac{1}{2}mg}{\frac{7}{48}ml^2}$$

counter clockwise (note sign).

$$\boxed{\alpha = \frac{12}{7}\frac{k}{m}\left(\sqrt{5} - \sqrt{3}\right) - \frac{6}{7}\frac{g}{l}}$$ ⟲+

Ask yourself "Does this make sense?" if $k\uparrow$ then $\alpha\uparrow$ too, if m or $g\uparrow$ then $\alpha\downarrow$

6/60 find support at O when Bar released, neglect bracket

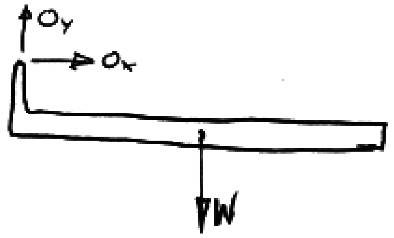


$m = 300\ kg$

$I_G = \frac{1}{12} m l^2 \qquad I_O = I_G + m d^2 \qquad d = \sqrt{2^2 + 0.9^2}$

$\therefore\ I_O = \left(\frac{1}{12}\right)(300)(16) + (300)\left(2^2 + (0.9)^2\right) = \underline{1843\ kg\,m^2}$

Free Body Diagram

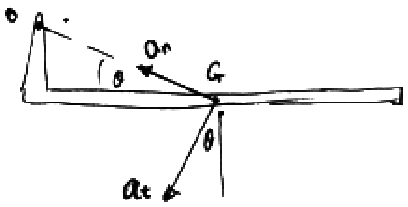


$\Sigma M_O = I_O \alpha$

$(W)(2) = 1843\ \alpha$

$\alpha = \frac{(300)(9.81)(2)}{1843} = 3.19\ rad\ s^{-2}$

Acceleration DIAGRAM



note $\vec{a}_n$ & $\vec{a}_t$ are $\perp$ & $\parallel$ to radius $\vec{OG}$

$\vec{a}_n = \vec{0}$ as $\omega = 0$ so $r\omega^2 = 0$

(because it has just been released

$a_t = r\alpha$

$= \sqrt{(2)^2 + (0.9)^2}\,(3.19) = 7\ m/s^2$

$\Sigma F_x = m a_x$

$O_x = (300)(7)\frac{(0.9)}{\sqrt{(2)^2 + (0.9)^2}} = 862\ N$

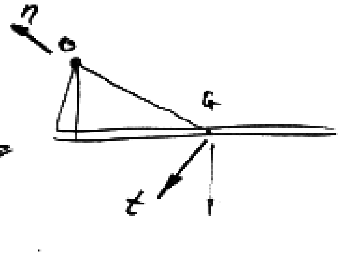
$\Sigma F_y = m a_y$

$W - O_y = m a_y$

$O_y = (300)(9.81) - (300)(7)\frac{(2.0)}{\sqrt{(2)^2 + (0.9)^2}} = 1028\ N$

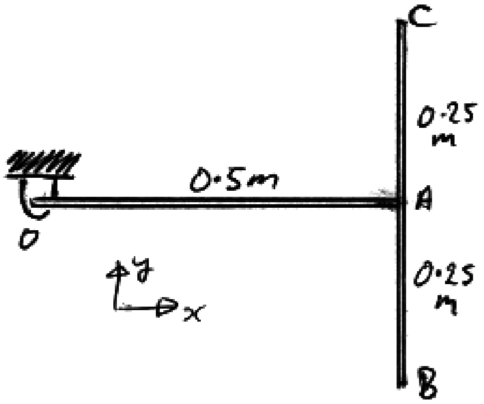
$\|\vec{O}\| = \sqrt{(1028)^2 + (862)^2} = \underline{\underline{1341\ N}}$

Note, it would have been appropriate also to use a normal & tangential coordinate system as shown

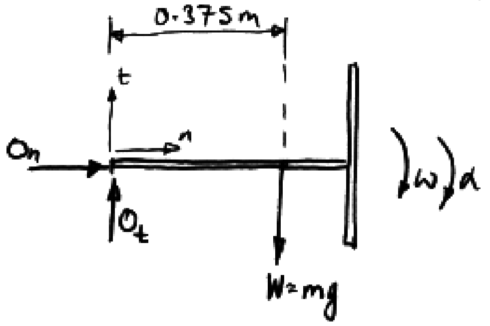


6/63 Each Bar has mass of 8kg & the 2 are welded into a "T" shape.

find force @ O when bar is in position shown, and given that $\omega = 4$ rad/s at that instant.



first draw free Body diagram



need to calculate location of centre of mass (use x-y as shown above)

$$\bar{x} = \frac{(8)(0.25)+(8)(0.5)}{16} = 0.375\,m$$

$$\bar{y} = \frac{(8)(0)+(8)(0)}{16} = 0$$

I_G for bar is $\frac{1}{12}ml^2$

I_o for OA-Bar is $\left(\frac{1}{12}\right)(ml^2)+(m)(0.25)^2 = 0.6667\ kgm^2$

I_o for BC-Bar is $\left(\frac{1}{12}\right)(ml^2)+(m)(0.5)^2 = 2.1667\ kgm^2$

I_o total $\overline{2.833\,kgm^2}$ ← Sum

$$\Sigma M_o = I_o\alpha$$

$$(mg)(0.375) = (2.833)\alpha \Rightarrow \boxed{\alpha = \frac{(16)(9.81)(0.375)}{(2.833)} = 20.78\ rads^{-2}}$$

NOTE m = total mass = 16 kg

then, to get forces

$$\Sigma F_t = ma_t$$

$$W - O_t = mr\alpha$$

$$O_t = W - mr\alpha = mg - mr\alpha = (16)(9.81) - (16)(0.375)(20.78)$$

$$O_t = 32.28\ N$$

$\Sigma F_n = ma_n$

$$O_n = -mr\omega^2$$

$$= -16(0.375)(4)^2$$

$$O_n = 96\ N$$

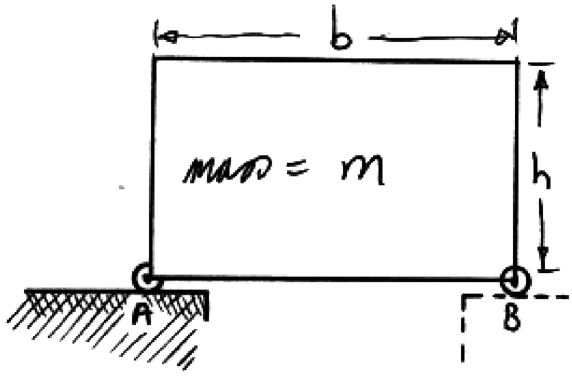
then $\|\vec{O}\| = \sqrt{O_n^2 + O_t^2}$

$$= \sqrt{(96)^2 + (32.3)^2} =$$

$$\boxed{|\vec{O}| = 101.28\,N}$$

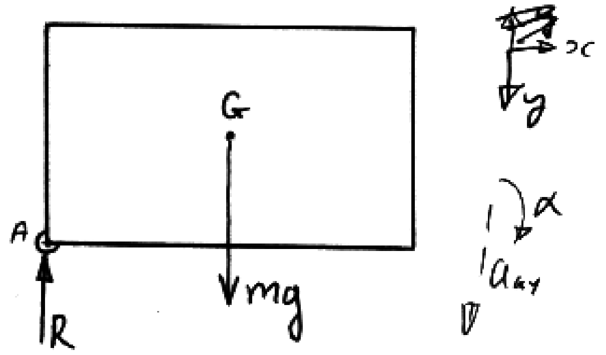
6/102

Support at B suddenly removed, find reaction force at A at that instant



$I_G = \frac{m}{12}(h^2+b^2)$ for rectangle.

draw free body diagram
assume roller smooth
$\Rightarrow$ no x-force at A



$\Sigma F_x = m a_{G_x} = 0$

$\boxed{\therefore a_{Gx} = 0}$

$\Sigma F_y = m a_{Gy}$

$\Rightarrow \boxed{mg - R = m a_{Gy}}$ (1)

$\Sigma M_G = I_G \alpha$

$\boxed{(R)\left(\frac{b}{2}\right) = \frac{m}{12}(h^2+b^2)\alpha}$ (2)

(1) & (3) $\Rightarrow mg - R = \frac{b\alpha m}{2}$ (4)

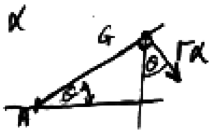
(4) & (2) $\cancel{m}g - \frac{\cancel{m}}{6b}(h^2+b^2)\alpha = \frac{b\alpha \cancel{m}}{2}$

$\alpha\left(\frac{b}{2} + \frac{h^2+b^2}{6b}\right) = g$

$\boxed{\alpha = \frac{g6b}{h^2+4b^2}}$ (5)

Note $\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}$ — accln of G relative to A

also $a_{G/A} = r\alpha = \sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{h}{2}\right)^2}\,\alpha$



$\cancel{a_{Gx}}^{0} = a_{Ax} + a_{G/A_x}$

$a_{Ax} = \frac{1}{2}\sqrt{b^2+h^2}\left[\frac{h}{\sqrt{b^2+h^2}}\right]\alpha$ — $\sin(\theta)$

$\boxed{a_{Ax} = \frac{h\alpha}{2}}$ (6)

$a_{Ay} = 0$ and $a_{Gy} = \cancel{a_{Ay}}^{0} + a_{G/A_y}$

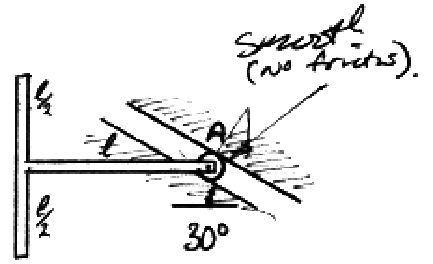
so $\boxed{a_{Gy} = \frac{mg-R}{m} = \frac{1}{2}\sqrt{b^2+h^2}\,\frac{b}{\sqrt{b^2+h^2}}\,\alpha = \frac{b\alpha}{2}}$ (3)

Subst. (5) into (6)

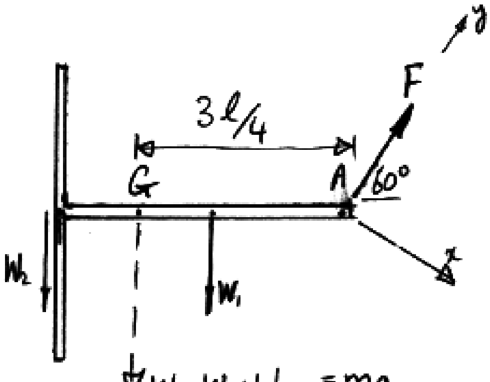
$\Rightarrow a_A = a_{Ax} = \frac{h\alpha}{2} = \frac{h}{2}\left(\frac{6gb}{h^2+4b^2}\right) = \frac{3gbh}{h^2+4b^2}$

$\boxed{a_A = \frac{3gbh}{h^2+4b^2}}$

6/106 | mass of T shaped body is m
it is released in position shown,
find initial accln of point A



Draw Free Body diagram



$W = W_1 + W_2 = mg$

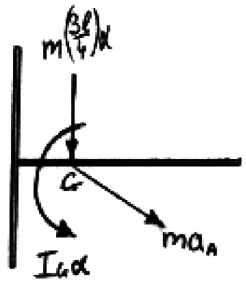
we can think of W_1 & W_2 or easier to think of $(W_1 + W_2) = W$ acting at G
(see prob 6/63 for more info).

$W_1 = W_2 = \frac{1}{2}mg$

$$I_G = \frac{1}{12}\frac{m}{2}l^2 + \frac{m}{2}\left(\frac{l}{4}\right)^2 + \frac{1}{12}\frac{m}{2}l^2 + \left(\frac{l}{4}\right)^2\frac{m}{2}$$

$$\boxed{I_G = ml^2\left(\frac{7}{48}\right)}$$

accln diagram



take moments about A

$$\sum M_A = I_G\alpha + ma_G d$$

$$(mg)\left(\frac{3l}{4}\right) = ml^2\left(\frac{7}{48}\right)\alpha + ma_G d$$

note, we can think of $\vec{a}_G$ as having 2 components
the acceleration of roller A + the acceleration due to rotation about A. $\vec{a}_A$ will be parallel to the guide.

so we have

$$mg\left(\frac{3l}{4}\right) = ml^2\left(\frac{7}{48}\right)\alpha + m\left(\frac{3l}{4}\alpha\right)\left(\frac{3l}{4}\right) + m\,a_A\left(\frac{3l}{4}\right)\left(\frac{1}{2}\right)$$

also

$$\sum F_x = ma_{G_x} \Rightarrow mg\sin(30°) = m\left(a_A + \frac{3l}{4}\alpha\,\frac{1}{2}\right)$$

Solve simultaneous Equations

$$\Rightarrow \alpha = \frac{108}{109}\frac{g}{l}$$

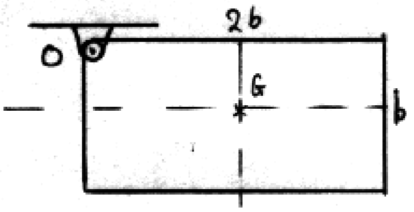
$$a_A = \frac{14}{109}g$$

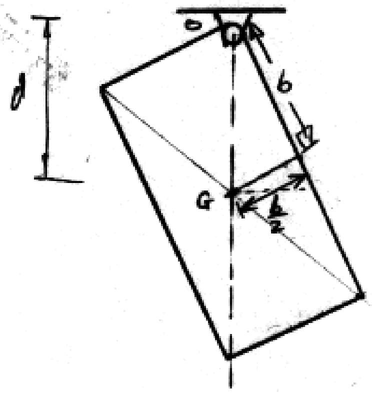
(6/117)

max angular velocity occurs WHEN CENTER of GRAVITY REACHES ITS LOWEST POINT

then

change in potential energy = change in kinetic energy.





change in height of G is

$$\frac{b}{2} - \sqrt{b^2 + \frac{b^2}{4}} = b\left(\frac{1-\sqrt{5}}{2}\right)$$

What is I_o for plate?

$$I_G = \frac{m}{12}\left[b^2 + (2b)^2\right] = \frac{5mb^2}{12}$$

Parallel axis theorem...

$$I_o = I_G + md^2 = \frac{5mb^2}{12} + m\left(b^2 + \left(\frac{b}{2}\right)^2\right)$$

$$= \frac{5mb^2}{12} + \frac{5mb^2}{4} = \frac{20mb^2}{12}$$

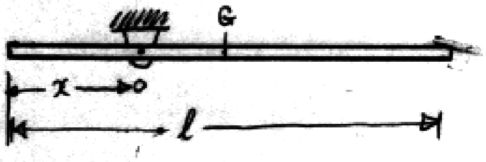
since initial kinetic energy is zero

$$-mgb\left(\frac{1-\sqrt{5}}{2}\right) = \frac{1}{2} I_o \omega^2 = \frac{1}{2}\,\frac{20mb^2}{12}\,\omega^2$$

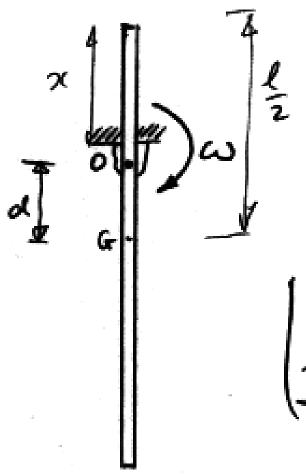
$$\omega^2 = \frac{-\cancel{m}g\cancel{b}\left(\frac{1-\sqrt{5}}{2}\right)}{\frac{10\cancel{m}b^{\cancel{2}}}{12}}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{b}}\,(0.861)$$

released from rest



max ang vel. attained



$\boxed{\text{find } x \text{ for max } \omega}$

Look at conservation of energy

$$\underbrace{mg\Delta h}_{\text{potential}} + \underbrace{\Delta K.E.}_{\text{kinetic en}} = 0$$

i.e. $mg\Delta h = -\frac{1}{2}I_o\omega^2$ ⊛

$\Delta h = -\left(\frac{l}{2} - x\right)$

$I_o = ?$ $\quad I_G = \frac{1}{12}ml^2$ for slender rod

parallel axis theorem...

$I_o = I_G + md^2$ and $d = \left(\frac{l}{2} - x\right)$

$\therefore \underline{I_o = \frac{1}{12}ml^2 + m\left(\frac{l}{2} - x\right)^2}$

rearrange ⊛

$$\omega^2 = \frac{-mg\Delta h}{\frac{1}{2}\left[\frac{1}{12}ml^2 + m\left(\frac{l}{2} - x\right)^2\right]} = \frac{2g\left(\frac{l}{2} - x\right)}{\frac{l^2}{12} + \frac{l^2}{4} - lx + x^2}$$

differentiate w.r.t. x, set to zero.

$$\frac{d\omega^2}{dx} = -\cancel{2g}\left(\frac{l^2}{12} + \frac{l^2}{4} - lx + x^2\right) - \cancel{2g}\left(\frac{l}{2} - x\right)\left(-l + 2x\right) = 0$$

$$-\frac{l^2}{3} + lx - x^2 + \frac{l^2}{2} - \cancel{lx} \cancel{2lx} + 2x^2$$

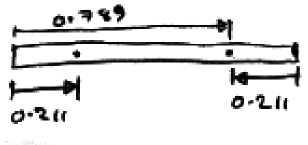
$$+\frac{l^2}{6} - lx + x^2 = 0$$

$$\Leftrightarrow \left(\frac{x}{l}\right)^2 - \left(\frac{x}{l}\right) + \frac{1}{6} = 0 \Rightarrow \frac{x}{l} = \frac{-1 \pm \sqrt{1 - \frac{4}{6}}}{2} = \begin{matrix} 0.789 \\ \text{or} \\ 0.211 \end{matrix}$$

note that

$0.789 + 0.211 \approx 0$

this SHOULD make sense TO you as we are looking at a uniform ROD.

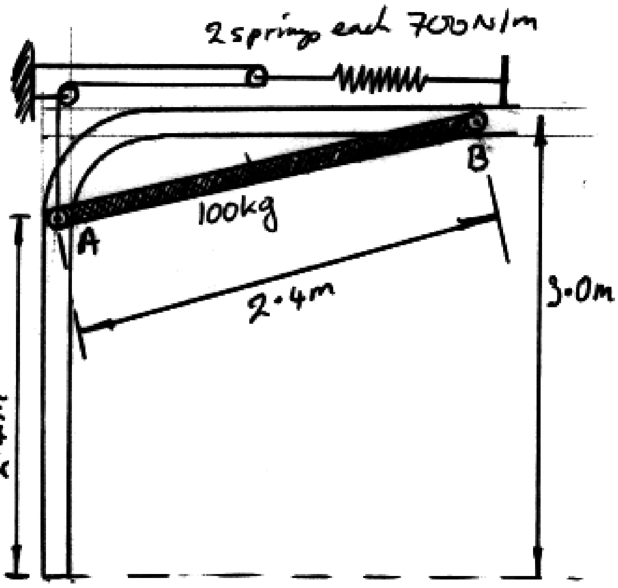


can use $x = 0.211\, l$ to find ω_{max}

$$\omega = \sqrt{\frac{2g\left(\frac{l}{2} - x\right)}{\frac{l^2}{3} - lx + x^2}} = 1.861\sqrt{\frac{g}{l}}$$

other value for x gives same ω, but in opposite dirxn of rotation

6/140 In position shown, spring has no stretch find velocity of A as it strikes GARAGE floor.



note, as door hits floor, it has no angular velocity since at that moment it is constrained to move straight down.

initially zero velocity

$\therefore \underbrace{\Delta T}_{\text{kinetic}} + \underbrace{\Delta V_g}_{\text{gravity}} + \underbrace{\Delta V_e}_{\text{elastic}} = 0$

$\Delta V_g = mg\Delta h = -(100)(9.81)\underbrace{\left[\left(2.4 + \frac{0.6}{2}\right) - \left(\frac{2.4}{2}\right)\right]}_{1.5\,m} = -1471.5\ J$

$\Delta V_e = \underset{\uparrow\ 2\ \text{springs}}{2}\left(\frac{1}{2}kx^2\right)$,

A travels 2·4m, but since there is a pully as shown, spring stretches half this $= 1.2\ m$

$\therefore \Delta V_e = 2\left(\frac{1}{2}(700)(1.2)^2\right) = 1008\ J$

$\Delta T = \frac{1}{2}mv^2 = -\Delta V_g - \Delta V_e = 1471.5 - 1008 = 463.5$

$v^2 = \frac{(463.5)(2)}{m} = \frac{(463.5)(2)}{100}$

$\underline{\underline{v = 3\ m/s}}$

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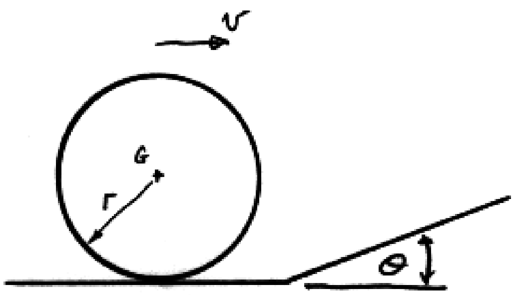
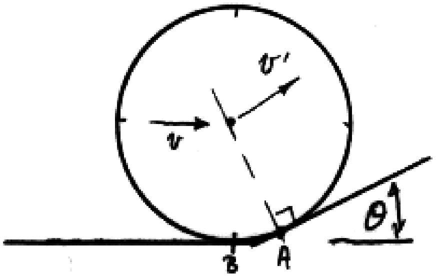
What is v', new velocity once disc starts up incline?

find fraction of energy lost for $\theta = 10°$



Look @ moment of impact:

impacts @ point A



$\Delta H_A = 0$ no change in angular momentum about A

(force @ A has no moment about A, reaction force @ B & weight cancel out

$$I_G \omega + m v r \cos\theta = I_A \frac{v'}{r} \qquad I_G = \tfrac{1}{2} m r^2 \quad I_A = \tfrac{1}{2} m r^2 + m r^2 = \tfrac{3}{2} m r^2$$

$$\tfrac{1}{2} m r^2 \frac{v}{r} + m v r \cos\theta = \tfrac{3}{2} m r^2 \frac{v'}{r}$$

$$\therefore \quad \underline{v' = \frac{v}{3}(1 + 2\cos\theta)}$$

N.B. if $\theta = 0$ $v' = \frac{v}{3}(1 + (2)(1)) = v$ useful check to see answer is correct

$$\% \Delta K.E. = \frac{\frac{1}{2} I_B \omega^2 - \frac{1}{2} I_A \omega'^2}{\frac{1}{2} I_B \omega^2} \qquad I_A = I_B,$$

$$= 1 - \left(\frac{\omega'}{\omega}\right)^2 = 1 - \left(\frac{v'}{v}\right)^2$$

$$= 1 - \left(\frac{1 + 2\cos\theta}{3}\right) \quad \text{if } \theta = 10° \text{ we get...}$$

$$1 - \frac{1 + 2\cos(10°)}{3}$$

$$= 0.0202$$