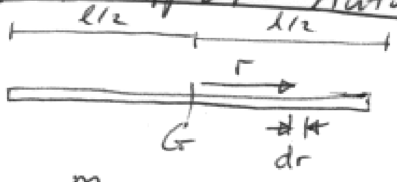
MEEN 2001 2nd Yr App. Dyn Autumn 2004 Dr. M. Conry Q.4

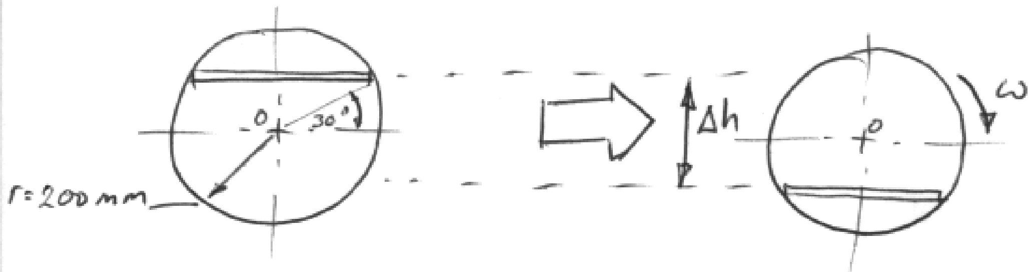
Show

$I_G = \frac{1}{12} m l^2$



$I_G = \int_{-\frac{l}{2}}^{\frac{l}{2}} r^2 \, dm \qquad dm = \frac{m}{l} dr$

$\therefore I_G = \frac{m}{l} \int_{-l/2}^{l/2} r^2 \, dr = \frac{m}{l} \left[\frac{r^3}{3} \right]_{-l/2}^{+l/2} = \frac{m}{l} \left(\frac{l^3}{24} + \frac{l^3}{24} \right) = \frac{1}{12} m l^2$



rod changes potential energy
Disk Doesn't
Both rod & Disk change kinetic energy

rod = 2kg = m Disk = 6kg = 3m

Smooth bearing @ O → conservation of Energy

$\cancel{T_1}^{\,0} + V_{1-2} = T_2 \qquad V_{1-2} = mg\Delta h = mg\,2(r\sin(30^\circ)) = mgr$

$T_2 = \frac{1}{2} I_o \omega^2$

$I_o = I_{o\,Disk} + I_{o\,rod} \qquad I_{o\,Disk} = \frac{1}{2} M_{Disk} r^2 = \frac{3r^2}{2} m$

$\boxed{l = 2r\cos 30^\circ = \sqrt{3}\, r}$

$I_{o\,ROD} = \frac{1}{12} m l^2 + \frac{mr^2}{4}$ Parallel axis Theorem

$= \frac{m 3 r^2}{12} + \frac{mr^2}{4}$

$= \frac{1}{2} m r^2$

$\therefore I = \left(\frac{3}{2} + \frac{1}{2} \right) m r^2 = \underline{\underline{2mr^2}}$

$V_{1-2} = \frac{1}{2} I_o \omega^2 \Rightarrow \omega^2 = \frac{2V_{1-2}}{I_o} = \frac{\cancel{2}\cancel{m} g \cancel{r}}{\cancel{2}\cancel{m} r^{\cancel{2}}} = \frac{g}{r}$

$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{0.2}} = 7 \text{ rad/s}$