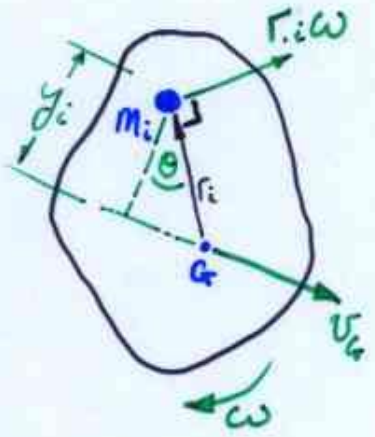
KINETIC ENERGY

GENERAL PLANE MOTION



velocity of mass centre is v_G

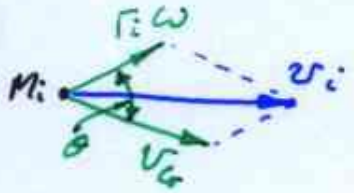
angular velocity is ω

Particle m_i has 2 components of vel

$\vec{v}_i = \vec{v}_G + \vec{r}_i \times \vec{\omega}$

vector addition

$v_i = \sqrt{v_G^2 + (r_i\omega)^2 + 2 v_G r_i \omega \cos\theta}$



Cosine Rule

θ is angle between $r_i\omega$ & v_G

$\therefore T_i = \frac{1}{2} m_i v_i^2$

$= \frac{1}{2} m_i \left[v_G^2 + r_i^2\omega^2 + 2 v_G r_i \omega \cos\theta \right]$ (from c.o.g)

Sum/integrate; 3rd term → zero by Defn of C. of G.

$\Rightarrow T = \frac{1}{2} m v_G^2 + \frac{1}{2}\omega^2 \int m_i r_i^2$ $\int r_i^2 dm$

$$\Rightarrow \boxed{T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2}$$

if you know a point that has instantaneous ~~instant~~ zero velocity, then you can also say

$$\boxed{T = \frac{1}{2} I_c \omega^2}$$

I_c moment of inertia about that point