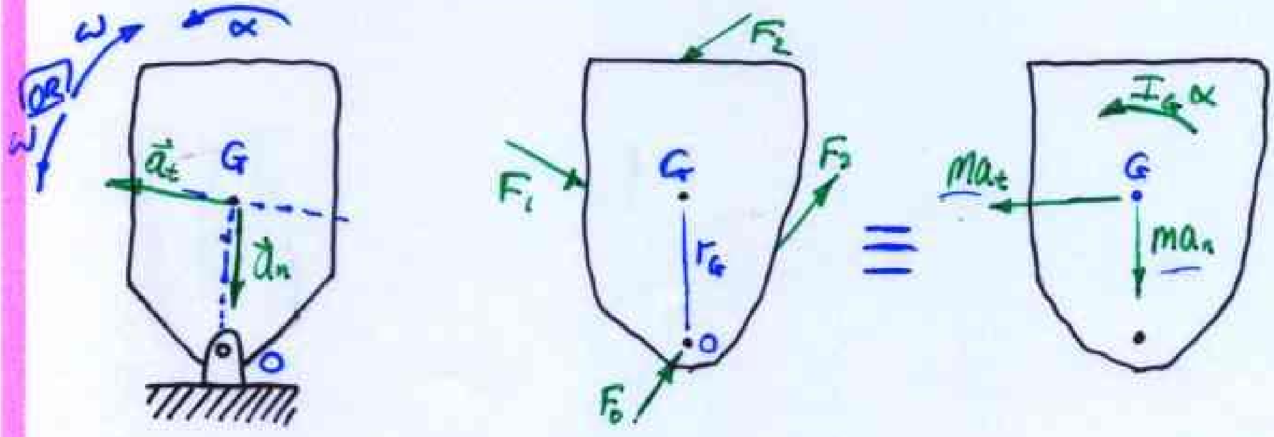
Plane Kinetics of Rigid Bodies

Last week we looked at TRANSLATION

THIS WEEK: <u>FIXED AXIS ROTATION</u>

ALL POINTS ON BODY MOVE ALONG CIRCLES WITH A common centre at the axis of rotation



$a_n = r_G\omega^2$

$a_t = r_G\alpha$

N.B. note that a force occurs at O

As before we can write

$$\sum \vec{F} = m\vec{a}_G \qquad \sum M_G = I_G\alpha$$

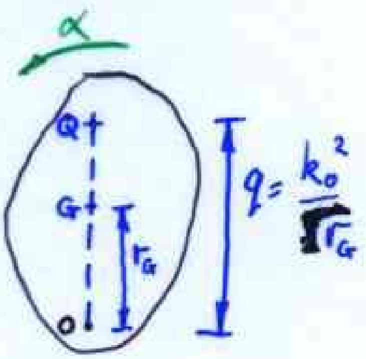
vector sum — combination — SCALAR BECAUSE 2D PROB

Often handier to sum moments about O

$\Rightarrow \sum M_o = I_o\alpha$ (derived last week)

Center of Percussion

combine resultant force ma_t and moment $I_G\alpha$ by moving ma_t to a parallel posn @ point Q



$$m\overbrace{r_G\alpha}^{a_t}q = I_G\alpha + mr_G\alpha r_G \qquad \sum M_o = 0$$

$$\Rightarrow q = \frac{k_o^2}{r_G}$$

← (radius of gyration)^2 about o

F2

Some useful relations:

ω_0 = orig value of ω
θ_0 = " " " " θ

if α is constant ... $\omega = \omega_0 + \alpha t$

[NOTE SIMILARITY to EQN'S FOR LINEAR ACCLN]

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

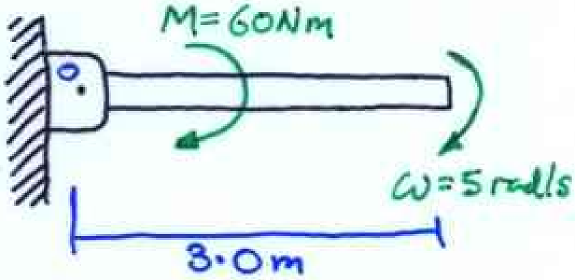
if α is not constant, more general expression needed

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \; ; \quad \omega = \frac{d\theta}{dt} \; ; \quad \alpha \, d\theta = \omega \, d\omega$$

AN EXAMPLE: Moment M = 60 Nm

20.0 kg slender rod rotates in vertical plane. $\omega = 5 \text{ rad s}^{-1}$

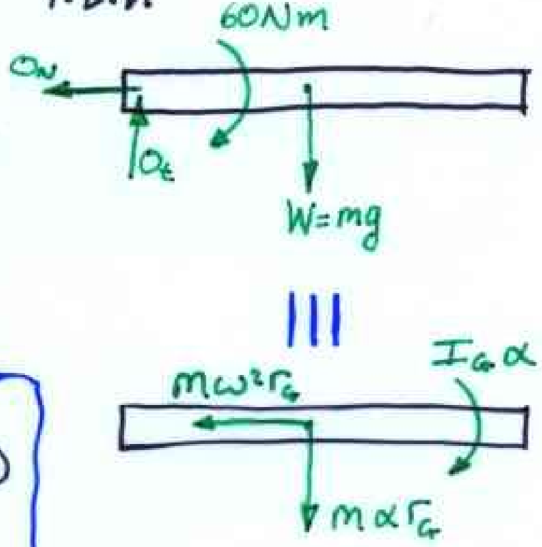
FIND α & components of forces @ O



3 UNKNOWNS O_N, O_t, α

EQUATIONS...

F.B.D.



$+\sum F_N = m a_N$

$$O_N = m r_G \omega^2 = (20)(1.5)(5)^2 = \underline{750 N}$$

$+\sum F_t = m a_t$

$$-O_t + mg = m\alpha r_G \quad (1)$$

[$\sum M_G = I_G\alpha \Rightarrow O_t r_G + M = I_G\alpha$

$$-O_t(1.5) - 60 = -\left[\tfrac{1}{12}(20)(3)^2\right]\alpha \quad (2)$$

be careful with signs.]

note: $I_G = \frac{1}{12} m l^2$ for rod

SOLVE BETWEEN (1) & (2)

$\Rightarrow \underline{O_t = 19.05 N}$

$\underline{\alpha = 5.905 \text{ rad s}^{-2}}$

F3

ALTERNATIVELY we could sum moments about O

$$\sum M_o = I_o \alpha$$

$$-60 - mgr_G = -I_o \alpha$$

$$\Rightarrow \alpha = \frac{60 + mgr_G}{I_o} =$$

$$I_o = \tfrac{1}{3} m l^2 = \frac{(20)(3)^2}{3} = 60$$

Note $I_o = \overbrace{\tfrac{1}{12} m l^2}^{I_G} + md^2$
$= \tfrac{1}{12} m l^2 + m\left(\tfrac{l}{2}\right)^2$
$= \tfrac{1}{3} m l^2$

$$\alpha = \left[\frac{60 + (20)(9.81)(1.5)}{60}\right] = 5.905 \text{ rad s}^{-2}$$

this is quick route to α. CAN FIND O_N, O_t later if needed.

1 Mg = 1 ton

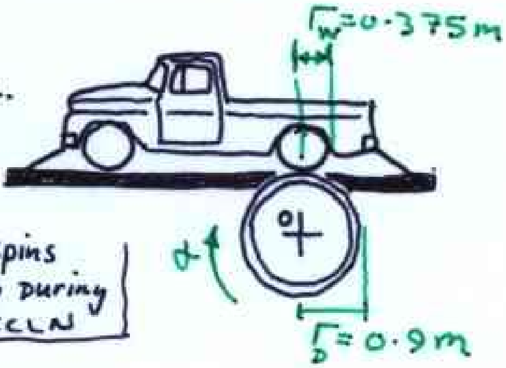
THIS DYNAMOMETER CAN SIMULATE ACCLN of 0.6 g for the Loaded truck.

MASS OF TRUCK is 2.8 Mg — 10^6 g = ton

FIND REQUIRED I_o of the DRUM if it spins freely during ACCLN

radius of wheel = 375 mm

" " DRUM = 900 mm



$$F = ma = \underbrace{(2800)}_{kg}(0.6)\underbrace{(9.81)}_{g\ ms^{-2}} = 16\,475 \text{ N} \quad \text{ACTING @ WHEEL}$$

$$a_t = (0.6)(9.81) = 5.884 \text{ ms}^{-2} \qquad a = r\alpha \text{ for wheel \& DRUM}$$

$$\text{so... } \alpha_{D_{rum}} = \frac{a}{r_D} = \frac{5.884}{0.9} = 6.538 \text{ rad s}^{-2}$$

$$\alpha_W = \frac{a}{r_W} = \frac{5.884}{0.375} = 15.69 \text{ rad s}^{-2}$$ — unnecessary

$$\sum M_o = I_o \alpha_D \quad \text{for DRUM}$$

$$\Rightarrow (F)(r_D) = I_o \alpha_D \Leftrightarrow I_o = \frac{(F)(r_D)}{\alpha_D}$$

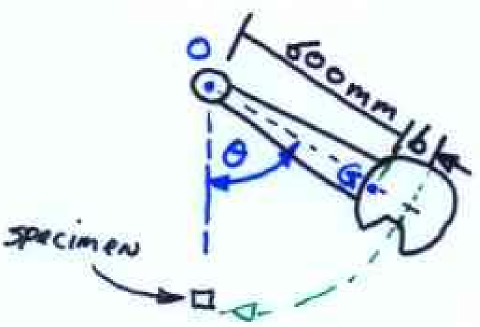
$$I_o = \frac{(16\,475)(0.9)}{(6.538)} = 2268 \text{ kg m}^2$$

6/48 Impact tester

mass of pendulum 34 kg

$k_o = 620$ mm radius of gyration

"b" has been chosen so that RXN at bearing O is minimised at impact.



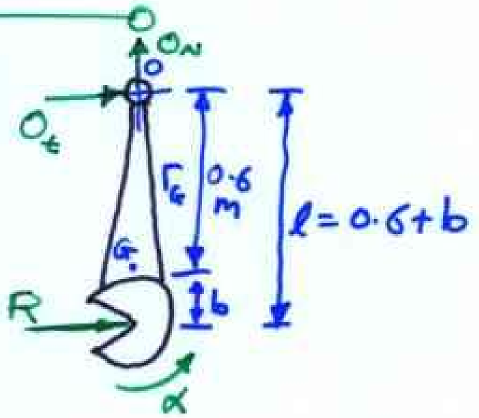
Find b & find force @ O when pendulum is released → $\theta = 60°$.

At impact, pendulum is vertical

specimen applies force $\vec{R}$

bearing applies O_t & O_N

gravity applies W (not shown)



$W = mg$

$\sum F_t = m a_{G_t}$

$(\overset{+}{\rightarrow}) \Rightarrow O_t + R = m\overline{a_{G_t}} = m\bar{r}\alpha$

$\boxed{R = m r_G \alpha - O_t}$ ①

Note $I_o = m k_o^2$

k_o = radius of gyration about O

$\sum M_o = I_o \alpha$

$\Rightarrow \boxed{(R)(\ell) = I_o \alpha}$ ②

subst from ① into ②

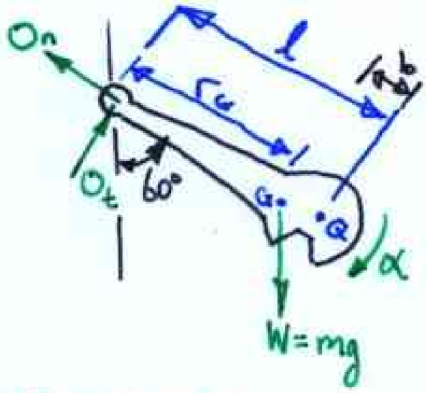
$(m r_G \alpha - O_t)(\ell) = I_o \alpha = m k_o^2 \alpha$

we want to minimise O_t, so set it to ZERO.

$\Rightarrow \cancel{m} r_G \cancel{\alpha}\, \ell = \cancel{m} k_o^2 \cancel{\alpha}$

$\Rightarrow \ell = \dfrac{k_o^2}{r_G}$ ... center of percussion

so $\ell = \dfrac{(0.620)^2}{(0.6)} = 0.6407$ m $\Rightarrow \boxed{b = \ell - 0.6 = 40.7 \text{ mm}}$



at moment of release

find O_t, O_N & $\|\vec{O}\|$

since it is just released $\Rightarrow \omega = 0$

$\sum F_N = ma_n$

$O_N - mg\cos 60° = mr\omega^2 = 0$ because $\omega = 0$ because just been released.

$\Rightarrow O_N = mg\cos(60°)$

$= (34)(9.81)(0.5)$

$\boxed{O_N = 166.77\,N}$

$\sum M_o = I_o\alpha$

$\Rightarrow (mg\sin 60°)r_G = mk_o^2\alpha$

$\Rightarrow \alpha = \dfrac{(34)(9.81)(0.866)(0.6)}{(34)(0.620)^2} = 13.26 \text{ rad s}^{-2}$

$\sum F_t = ma_{G_t}$

$\Rightarrow O_t - mg\sin 60° = -mr_G\alpha$

$\boxed{O_t = (34)(9.81)(0.866) - (34)(0.6)(13.26) = 18.35\,N}$

$\therefore \|O\| = \sqrt{(18.35)^2 + (166.77)^2} = \underline{167.8\,N} \approx O_N$

Note we could have used centre of percussion

$\sum M_Q = 0$ Prop of C. of. Percussion

$\Rightarrow mg(\sin 60°)b - O_t l = 0$; $l = 0.6407$ $b = 0.0407$

$O_t = \dfrac{mgb\sin 60°}{l} = \dfrac{(34)(9.81)(0.0407)(0.866)}{0.6407}$

$\boxed{O_t = 18.35\,N}$ again

GENERAL PLANE MOTION:

combination of translation & rotation

General eqns apply

$$\sum \vec{F} = m\vec{a}_G \qquad \sum M_G = I_G \alpha$$

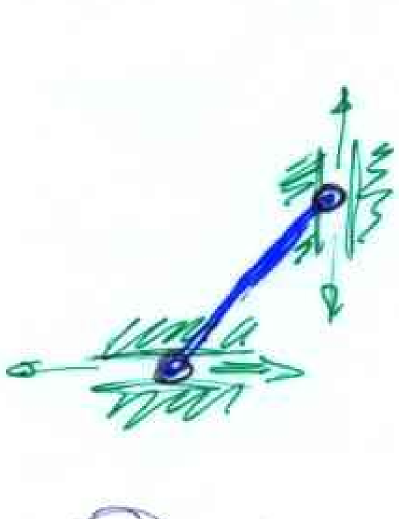
we repeat the alternative forms of MOMENT EQN

$$\sum M_P = I_G \alpha \pm m a_G d \qquad \sum M_P = I_P \alpha + \underbrace{\vec{\rho}_G \times m\vec{a}_P}_{\pm m a_P d}$$

SIGN DEPENDS ON RIGHT HAND RULE

STEPS:

- CHOOSE COORDINATE SYSTEM
- CHOOSE FORM OF MOMENT EQN
- IDENTIFY KINEMATIC CONSTRAINTS
  - ↳ incorporate into force, momentum eqns
- CHECK NO. of UNKNOWNS $\leq$ NO. of EQNS.
- CORRECTLY IDENTIFY BODY / SYSTEM
- USE ASSUMPTIONS CONSISTENTLY N.B.



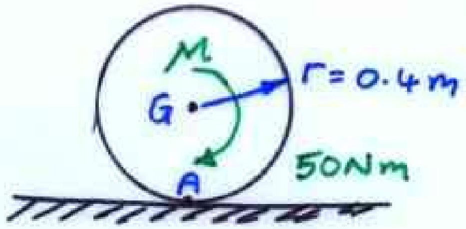
EXAMPLE

wheel, $W = 250N$

R. of gyr. $k_G = 0 \cdot 2m$

$50\,Nm$ torque applied,

find a_G if $\mu_s = 0 \cdot 3$ $\mu_k = 0 \cdot 25$
(friction coeffs)



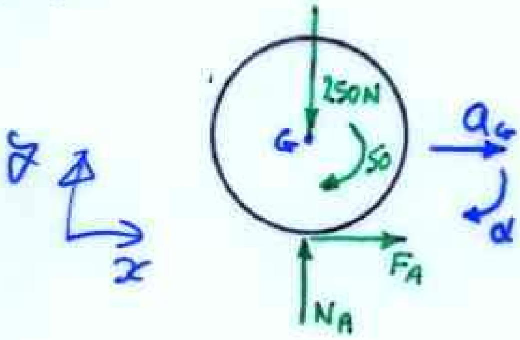
A is point of contact.

from dirxn of $M \Rightarrow \alpha$ clockwise $\Rightarrow a_G$ to right

$$I_G = mk_G^2 = \left(\frac{250}{9 \cdot 81}\right)(0 \cdot 2)^2 = \underline{1 \cdot 019}\ kg\,m^2$$

UNKNOWNS

N_A, F_A, a_G, α



EQNS of MOTION...

$\underline{\sum F_x = m a_{G_x}}$

$\Rightarrow F_A = m a_G$

$\underline{\sum F_y = m a_{G_y}} = 0$

$\Rightarrow N_A = 250N$ IN DIRXN SHOWN ↑

$\underline{\sum M_G = I_G \alpha}$

$\Rightarrow -50Nm + 0 \cdot 4 F_A = -I_G \alpha$ negative because of dirxn of α

Kinematics → Assume <u>NO SLIP</u>

$\Rightarrow \underline{a_G = 0 \cdot 4 \alpha}$

SOLVE SIMULTANEOUS EQNS

$\Rightarrow N_A = 250N$ $F_A = 100N$ $\alpha = 9 \cdot 81\ rad\,s^{-2}$ $a_G = 3 \cdot 92\,ms^{-2}$

Q) IS THIS VALID?
A) NO

i.e. is $F_A \leq \mu_s N_A$?

$100 \leq (0 \cdot 3)(250)$?

[UNTRUE] $100 \leq 75$? $\Rightarrow \boxed{NO} \Rightarrow$ false assumption
$\Rightarrow$ there IS SLIP

IF WHEEL SLIPS

$\Rightarrow F_A = \mu_k N_A$ use with 3 Eqns of Motion

we still have $\boxed{N_A = 250\,N}$

also $\boxed{F_A = \mu_k N_A = (0.25)(250) = 62.5\,N}$

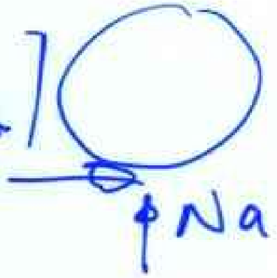
then $\boxed{a_{G_x} = \frac{F_A}{m} = \frac{62.5}{\left(\frac{250}{g}\right)} = 2.45\,ms^{-2}}$

$\Sigma M_G = I_G \alpha$

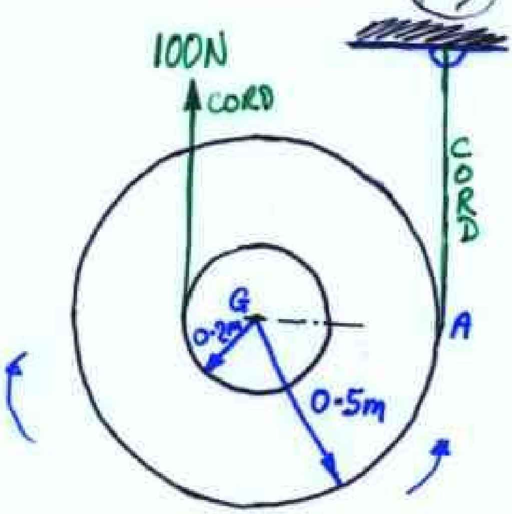
$$\alpha = \frac{50 - (0.4)(F_A)}{I_G} = \frac{50 - (0.4)(62.5)}{1.019}$$

$$\Rightarrow \boxed{\alpha = 24.5\ rad\,s^{-2}}$$

$f_{A_{max}} = \mu_s N_a$ (no slip condition)



Example: a spool has $m = 8$ kg radius of gyration $k_G = 0.35$ m if light cords are wrapped around inner & outer hubs as shown, find α for spool



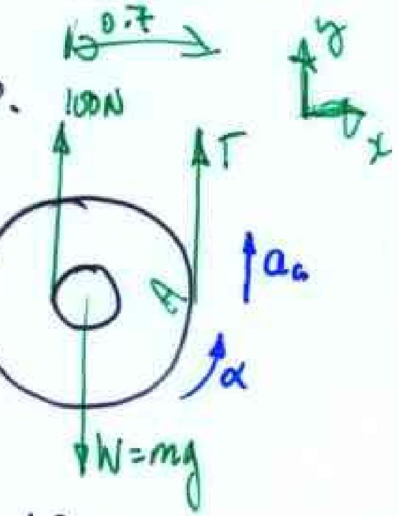
$\Sigma F_y = m a_{G_y}$ ①

(+↑) $\underline{T + 100N - mg = m a_G}$ N

$\Sigma M_G = I_G \alpha$

$\underline{-(100)(0.2) + (T)(0.5) = k_G^2 m \alpha}$ ② Nm

F.B.D.



Kinematics can relate a_G & α

assume spool "rolls without slipping" on cord at A

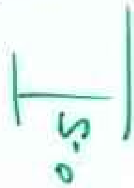
$\underline{a_G = -\alpha r = -0.5\alpha}$ ③

⇒ can solve our 3 eqns in 3 unknowns

Eqn ② rearranges to give

$I_G = k_G^2 m$

$$T = \frac{k_G^2 m\alpha + (100)(0.2)}{0.5} = 2k_G^2 m\alpha + 40$$



So ① becomes

$$\left[2k_G^2 m\alpha + 40\right] + 100 - mg = m(-0.5\alpha)$$

$$\Rightarrow \underline{\alpha} = \frac{-40 + mg - 100}{2k_g^2 m + \frac{1}{2}m} = \underline{-10.3\ \text{rad s}^{-2}}$$

then $\underline{a_G = 5.16\ \text{m s}^{-2}}$ & $\underline{T = 19.8\ \text{N}}$

To just find α (as required) there is a quicker way...

TAKE MOMENTS ABOUT A

$$\sum M_A = I_G\alpha - ma_G d$$ (get sign from R.H.R.)

$$-(100)(0.7) + mg(0.5) = I_G\alpha - ma_G(0.5)$$

$a_G = -0.5\alpha$ as before

$$\Rightarrow -70 + (8)(9.81)(0.5) = (8)(k_g^2)\alpha + (8)(0.5)\alpha(0.5)$$

$$\Rightarrow \underline{\alpha = -10.3 \text{ rad s}^{-2}}$$

Same as before

note sign $\Rightarrow$ α is in direction opposite to the one we sketched on Diagram

i.e. α is actually CLOCKWISE