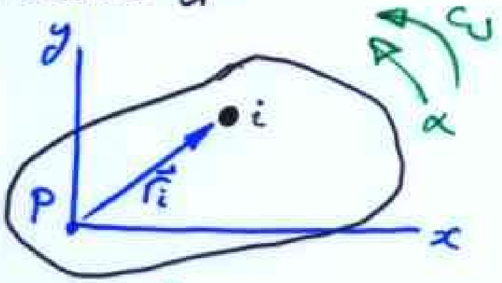
Another form is:

$$\sum \vec{M}_P = I_P \alpha + \vec{\rho} \times m \vec{a}_P$$

Here P is a point fixed to the body

$\vec{a}_P$ is accln of P

$\vec{\rho}$ is vector from P to mass centre G

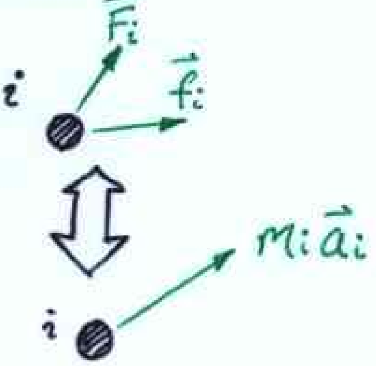


Consider small particle i

F_i = Resultant External force acting on particle

f_i = resultant internal force Due to interaction with adjacent particls

mass is m_i



Sum moments about P:

$$\vec{r}_i \times \vec{F}_i + \vec{r}_i \times \vec{f}_i = \vec{r}_i \times m_i \vec{a}_i$$

i.e. $\vec{M}_{P_i} = \vec{r}_i \times m_i \vec{a}_i$

in general, Point P can have an acceleration

$$\Rightarrow \vec{a}_i = \vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i$$

so $\vec{M}_{P_i} = m_i \vec{r}_i \times \left(\vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i \right)$

$$= m_i \left[\vec{r}_i \times \vec{a}_P + \vec{r}_i \times (\vec{\alpha} \times \vec{r}_i) - 0 \right] \qquad \boxed{\vec{r}_i \times \vec{r}_i = 0}$$