Last Lecture: LAST = Previous

INTRODUCED PLANE KINETICS OF RIGID BODIES

$$\sum \vec{F} = m\vec{a}_G \qquad \sum \vec{M}_G = \dot{\vec{H}}_G = \alpha I_G$$

(FOR 2-D CASE)

$I_G = \int r^2 dm$ moment of inertia

$= \rho \int r^2 dV$ if ρ is constant throughout volume

Parallel axis theorem: $I = I_G + md^2$

Radius of Gyration: $k = \sqrt{\frac{I}{m}} \iff I = k^2 m$

Consider alternative forms of moment eqn..

$$\sum \vec{M}_P = \dot{H}_G + \vec{\rho} \times m\vec{a}_G$$

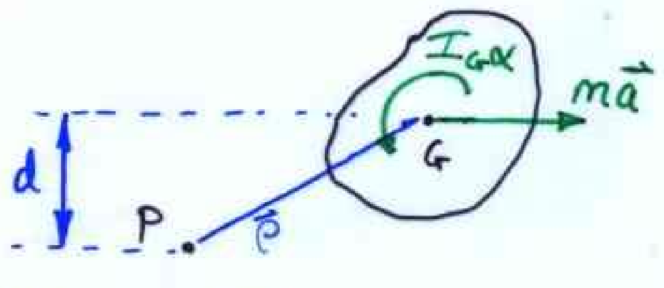
$\vec{\rho}$ is vector $\vec{PG}$

or 2D:

$$\sum M_P = I_G \alpha \pm m a_G d$$

$d = \perp$ dist between P & vector of accln

sign depends on right hand rule.



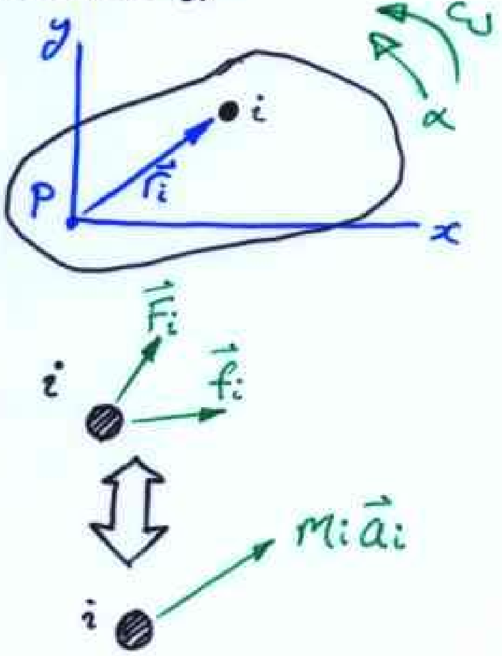
Here $\sum M_P = +I_G\alpha - mad$ NOTE sign

P is a nonaccelerating point

Another form is:

$$\sum \vec{M}_P = I_P \alpha + \vec{\rho} \times m \vec{a}_P$$

Here P is a point fixed to the body
$\vec{a}_P$ is accln of P
$\vec{\rho}$ is vector from P to mass centre G



Consider small particle i

F_i = Resultant External force acting on particle

f_i = resultant internal force Due to interaction with adjacent particles

mass is m_i

Sum moments about P:

$$\vec{r}_i \times \vec{F}_i + \vec{r}_i \times \vec{f}_i = \vec{r}_i \times m_i \vec{a}_i$$

i.e. $\vec{M}_{P_i} = \vec{r}_i \times m_i \vec{a}_i$

in general, Point P can have an acceleration

$$\Rightarrow \vec{a}_i = \vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i$$

so $\vec{M}_{P_i} = m_i \, \vec{r}_i \times \left(\vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i \right)$

$$= m_i \left[\vec{r}_i \times \vec{a}_P + \vec{r}_i \times (\vec{\alpha} \times \vec{r}_i) - 0 \right]$$

$\boxed{\vec{r}_i \times \vec{r}_i = 0}$

2D problem ...So:

$\therefore \vec{r}_i = x_i\hat{\imath} + y_i\hat{\jmath} \qquad \vec{a}_P = a_{P_x}\hat{\imath} + a_{P_y}\hat{\jmath}$

$\vec{\alpha} = \alpha\hat{k} \qquad$ and $\qquad \vec{M}_{P_i} = M_{P_i}\hat{k}$

So $$M_{P_i}\hat{k} = m_i\Big\{(x_i\hat{\imath} + y_i\hat{\jmath})\times\left[a_{P_x}\hat{\imath} + a_{P_y}\hat{\jmath}\right] + (x_i\hat{\imath}+y_i\hat{\jmath})\times\left[\alpha\hat{k}\times(x_i\hat{\imath} + y_i\hat{\jmath})\right]\Big\}$$

$$M_{P_i}\hat{k} = m_i\Big\{-y_i\, a_{P_x} + x_i a_{P_y} + \alpha\underbrace{(x_i^2+y_i^2)}_{\text{note } x^2+y^2=r^2}\Big\}\hat{k}$$

if we let $m_i \rightarrow dm$ & integrate over mass m we get...

$$\underbrace{\sum M_P}_{\substack{\text{moment of external}\\ \text{forces about}\\ \text{point } P\\ \text{(internal moments}\\ \text{cancel out)}}} = -\underbrace{\left(\int_m y\,dm\right)}_{m y_G} a_{P_x} + \underbrace{\left(\int_m x\,dm\right)}_{m x_G} a_{P_y} + \underbrace{\left(\int_m r^2\,dm\right)}_{\substack{\text{M.O.Inertia}\\ \text{about } P\\ I_P}}\alpha$$

i.e. the integrals locate centre of mass

$$\boxed{\sum M_P = m\vec{\rho}\times\vec{a}_P + I_P\alpha} \qquad \boxed{\vec{\rho} = x_G\hat{\imath} + y_G\hat{\jmath}}$$

Say $P = G$ (center of mass)

$\Rightarrow \underline{\sum M_G = m\vec{0}\times\vec{a}_G + I_G\alpha = I_G\alpha}$

OR Say P has no accl'n

$\Rightarrow \sum M_P = m\vec{\rho}\times\vec{0} + I_P\alpha$

$\underline{\sum M_P = I_P\alpha}$

Equations of Motion

TRANSLATION: i.e. all parts of a body undergoing TRANSLATION have the same acceleration.

$\Rightarrow$ no rotation about c.o.gravity

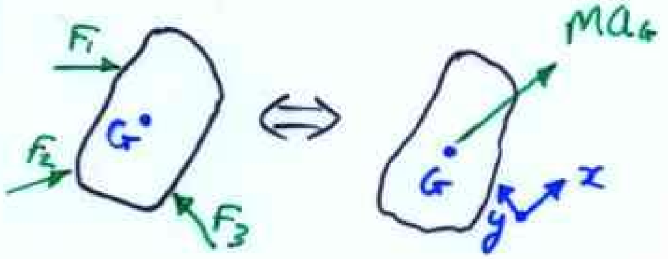
$\Rightarrow$ $\boxed{\Sigma M_G = 0}$ and of course: $\boxed{\Sigma \vec{F} = m\vec{a}_G}$

TRANSLATION may be RECTILINEAR or CURVILINEAR

Rectilinear Translation:

all points move in straight lines

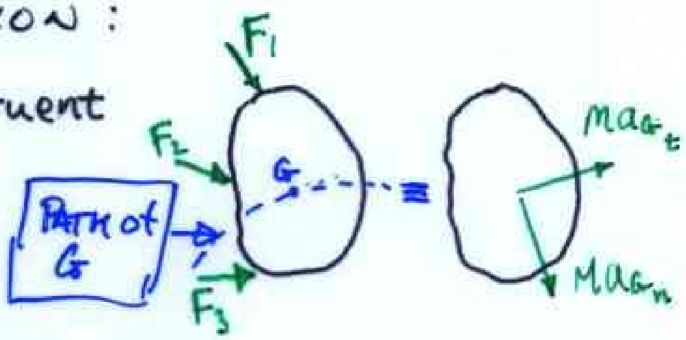
CONVENIENT TO PUT x-axis parallel to $\vec{a}$



Curvilinear Translation:

all points move on congruent curved paths

convenient to use components normal & tangential to path of G



$$\Sigma M_G = 0, \quad \Sigma F_n = m a_{G_n}, \quad \Sigma F_t = m a_{G_t}$$

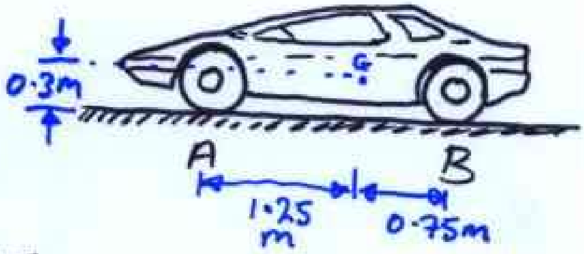
Note that other points can be used as moment centres

Example

car has mass 2000 kg

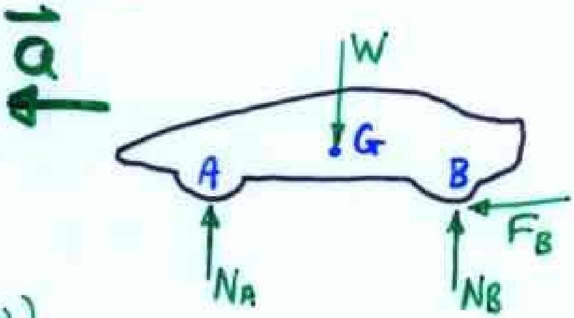
REAR WHEEL DRIVE

CALCULATE acc'n if rear WHEELS ARE SLIPPING & FRONT ROTATE FREELY



coeff of kinetic friction $\mu_k = 0.25$

DRAW FREE BODY DIAGRAM



$W = mg = (2000)(9.81) = 19.62\ kN$

$F_B = \mu_k N_B \qquad \sum F_y = m a_y = m(0)$

$+\uparrow \sum F_y = 0 \Rightarrow N_A + N_B - W = 0 \qquad (1)$

$\overset{+}{\leftarrow} \sum F_x = m a_{G_x} \Rightarrow \overbrace{\mu_k N_B}^{F_B} = 2000\, a_{G_x} \qquad (2)$

$\circlearrowleft + \sum M_G = 0 \Rightarrow -1.25 N_A + 0.75 N_B - (0.3)\mu_k N_B = 0 \qquad (3)$

Substitute (1) into (3) for ~~NB~~ NA

$$\Rightarrow -1.25\left[W - N_B\right] + 0.75 N_B - 0.3\mu_k N_B = 0$$

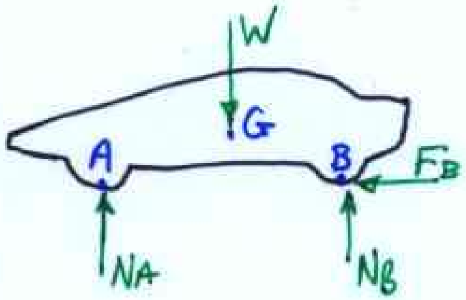
$$\Longrightarrow N_B = \frac{1.25W}{1.25 + 0.75 - 0.3\mu_k} = \frac{(1.25)(2000)(9.81)}{1.25 + 0.75 - (0.3)(0.25)} = 12\,740 N$$

$$a_{G_x} = \frac{\mu_k N_B}{m} = \frac{(0.25)(12\,740)}{(2000)} = \underline{1.59\ ms^{-2}}$$

$$\frac{F_B}{m}$$

There was an SLIGHTLY easier way to solve problem

TAKE MOMENTS ABOUT POINT A
then $\vec{N}_A$ & $\vec{F}_B$ HAVE zero moment



$$\circlearrowleft + \sum M_A = m a_G d$$

$$-W(1.25) + N_B(2.0) = +m\,a_{Gx}(0.3)$$

using ② from before i.e. $N_B = \dfrac{2000\,a_{Gx}}{\mu_k}$

gives $-W(1.25) + \left(\dfrac{2000\,a_{Gx}}{\mu_k}\right)(2.0) = +(m\,a_{Gx})(0.3)$

$$\Rightarrow a_{Gx} = \frac{W(1.25)}{\left[\dfrac{(2000)(2.0)}{\mu_k}\right] - (2000)(0.3)} = \frac{(2000)(9.81)(1.25)}{\dfrac{(2000)(2.0)}{\mu_k} - (2000)(0.3)}$$

$\underline{a_{Gx} = 1.5925 \text{ ms}^{-2}}$

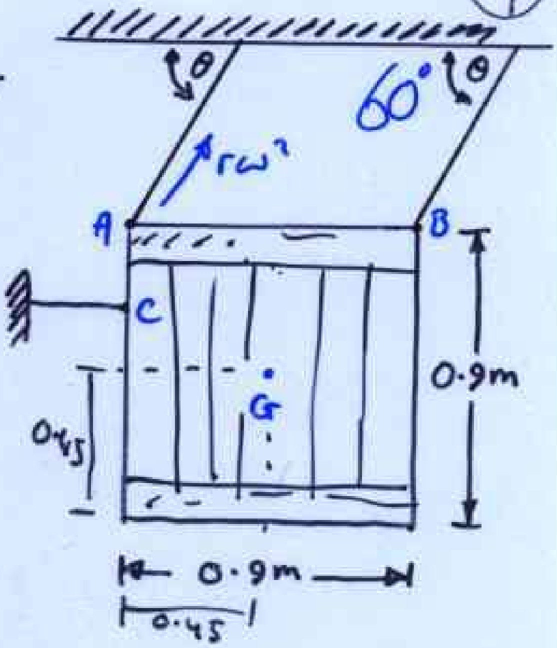
Note HOW a_{Gx} is INDEPENDENT of m (i.e. we have "2000" above & below)

a_{Gx} is a fn of "g" & geometry & μ_k.

6/33 mass of crate = 900kg

Rope at C is cut. Find tension in cable at A in instant immediately after. Solve using 3 eqns, and using just 1.

$\theta = 60°$



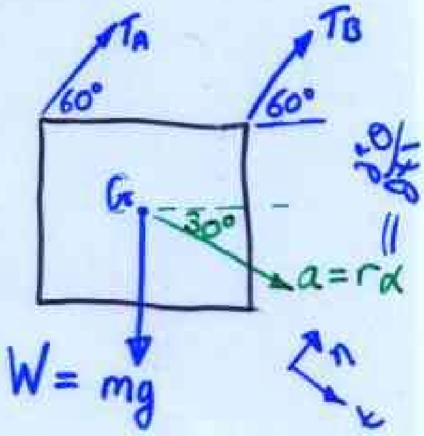
Just after rope cut, crate is still at rest.

$\Rightarrow \omega = 0$ so $r\omega^2$ component of acceleration is zero

$\dot{\theta}$

$\Rightarrow a = r\ddot{\theta}$ and is tangential to path of travel. (as shown in green)

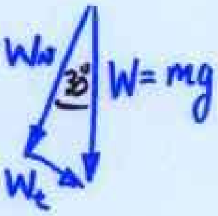
Use components n, t (normal, tangential)



$\sum F_n = ma_n = 0 = r\omega^2$

$\Rightarrow T_A + T_B - W_N = 0$

OR $T_A + T_B = mg\cos(30°)$ ⊛



$\left[\sum F_t = ma_t\right.$

$\Rightarrow W_t = ma_t \Rightarrow a = \dfrac{mg\sin(30°)}{m} = \dfrac{g}{2}\ \text{ms}^{-2}$

$a_n = 0$

$\Rightarrow a = a_t$

$\sum M_G = 0$

$\Rightarrow T_B\left[\dfrac{0.9}{\sqrt{2}}\sin(15°)\right] - \left[T_A\left(\dfrac{0.9}{\sqrt{2}}\right)\cos(15°)\right] = 0$

$\vec{W}$ has no moment about G

$\Rightarrow T_B = 3.7273\,T_A$

Substitute into 1st Expression ⊛

$T_A + 3.7273\,T_A = (900)(9.81)(\cos 30°)$

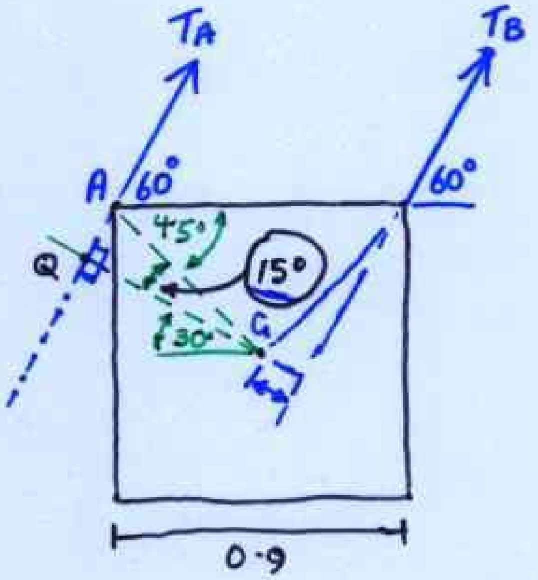
$\Rightarrow T_A = 1617\ \text{N}$

Note $|AG| = (0.9)\cos 45^\circ$

$$= \frac{0.9}{\sqrt{2}}$$

$$\Rightarrow |GQ| = \underbrace{\left(\frac{0.9}{\sqrt{2}}\right)\cos(15^\circ)}_{\text{moment arm of } \vec{T}_A}$$

MOMENT ARM of $\vec{T}_A$



more DIRECT APPROACH

take moment about P
i.e. intersection of $\vec{T}_B$ & $\vec{a}$

$$\boxed{\sum M_P = I_G \alpha + m a_G d}$$

$I_G\alpha = 0$ because TRANSLATION $\therefore \Rightarrow \underline{\alpha = 0}$

$m a_G d = 0$ because $d = 0$ due to location of P

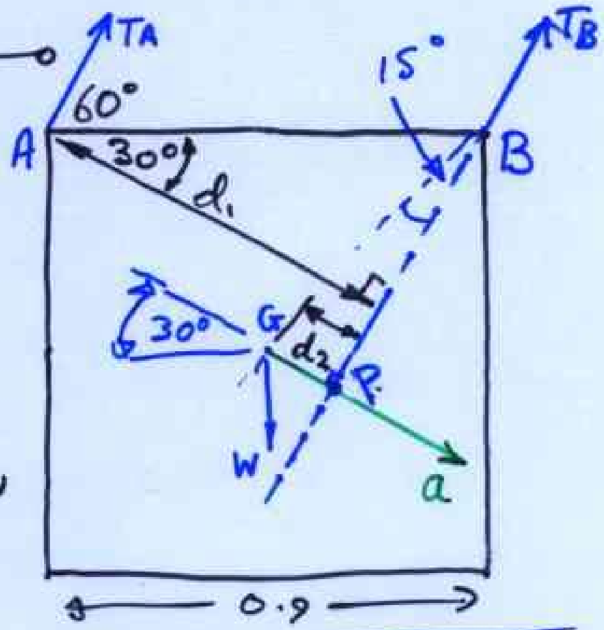
$\therefore \sum M_P = 0$

$\Rightarrow -T_A d_1 + W_N d_2 = 0$

$$\Leftrightarrow T_A = \frac{W_N d_2}{d_1}$$

$$T_A = \frac{\overbrace{(m)(g)\cos(30^\circ)}^{W_N}\overbrace{\left(\frac{0.9}{\sqrt{2}}\right)\sin(15^\circ)}^{d_2}}{\underbrace{(0.9)\cos(30^\circ)}_{d_1}}$$

$\underline{\underline{T_A = 1616\ N}}$



$d_1 = (0.9)(\cos 30^\circ)$

$d_2 = |PG|$

$|BG| = 0.9/\sqrt{2}$

$\Rightarrow |PG| = \frac{0.9}{\sqrt{2}}\sin(15^\circ)$

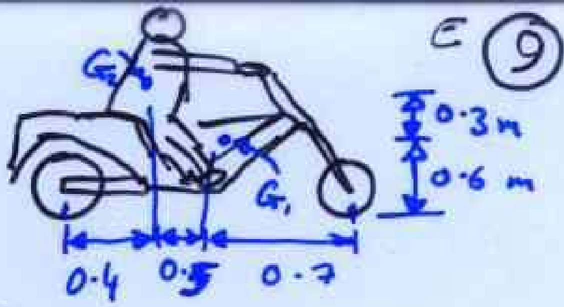
Motorcycle

MASS = 125 kg C.O.G = G_1

RIDER

MASS = 75 kg C.O.G = G_2

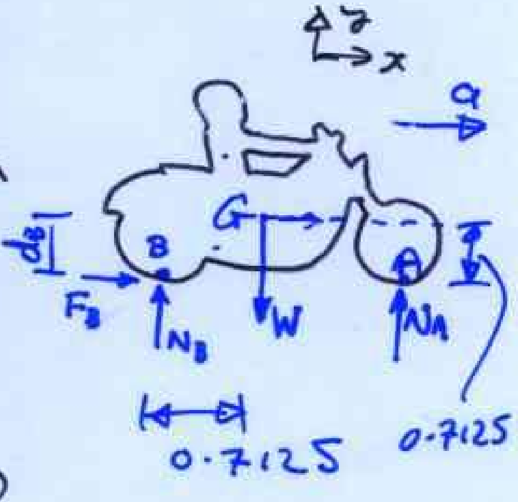
Coeff of friction wheel → ROAD = 0.8 = μ_s



CALCULATE if a wheelie is possible & if so @ WHAT ACCLN it will occur

Assume mass of wheels small & front wheel free to roll.

First, FIND centre of mass



$$x\text{-coord} = \frac{(125)(0.9)+(75)(0.4)}{200} = \underline{0.7125\,m}$$

$$y\text{-coord} = \frac{(125)(0.6)+(75)(0.9)}{200} = \underline{0.7125\,m}$$

Both relative to B

$$\Sigma F_x = m a_{Gx} \Rightarrow F_B = m a_G \quad ①$$

$$\Sigma F_y = m a_{Gy} = 0 \Rightarrow N_B + \cancel{N_A} - W = 0 \quad ②$$

$$\Sigma M_B = m a_G d_B \Rightarrow \underline{(N_A)(1.6)} - (W)(0.7125) = -m a_G (0.7125) \quad ③$$

IF A wheelie is about to occur ⇒ $N_A = 0$

② ⇒ $N_B = W = (200)(9.81)$

③ ⇒ $(\cancel{0.7125})(\cancel{200})(9.81) = +(\cancel{200})(a_G)(\cancel{0.7125})$

⇒ $a_G = 9.81\ ms^{-2}$

① ⇒ $F_B = m a_G = (200)(9.81)$

So we HAVE $F_B = (200)(9.81)$ & $N_B = (200)(9.81)$

BUT, $\mu_s = \underline{0.8}$ ⇒ MAX value of F_B is $(0.8)(N_B)$

∴ wheelie <u>CANNOT</u> occur.

i.e. WHEEL WILL SKID BEFORE A WHEELIE WILL OCCUR.

So... WHAT IS MAX ACCLN POSSIBLE?

JUST BEFORE WHEEL SPIN $\boxed{F_B = \mu_s N_B}$ $\mu_s = 0.8$

from ① $N_A = W - N_B = mg - N_B$

from ② $ma_G = F_B = \mu_s N_B$

then ③ $(N_A)(1.6) - W(0.7125) = -ma_G(0.7125)$

Becomes

$$(mg - N_B)(1.6) - mg(0.7125) = -\mu_s N_B (0.7125)$$

$$\Rightarrow N_B = mg\left[\frac{0.7125 - 1.6}{-1.6 + (0.8)(0.7125)}\right] = 1690N$$

$$\therefore F_B = \mu_s N_B = (0.8)(1690N) = 1352N$$

$$\therefore a_G = \frac{F_B}{m} = \frac{1352}{200} = \underline{6.76\ ms^{-2}}$$

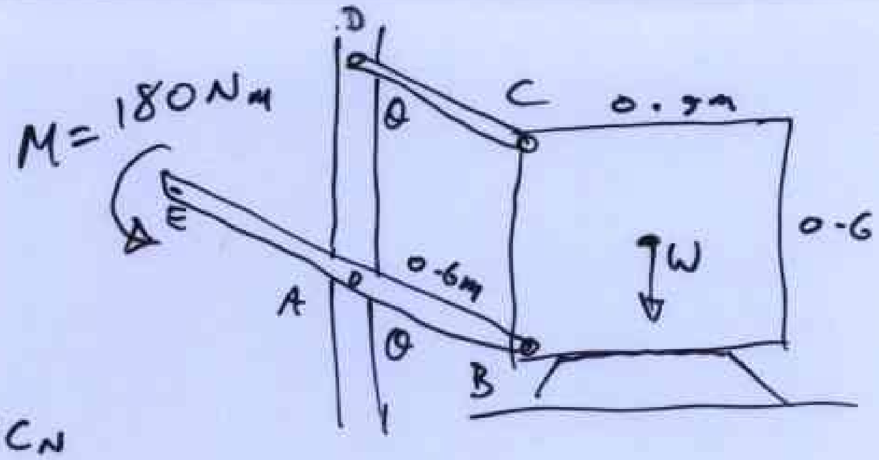
NOTE, AT THIS POINT

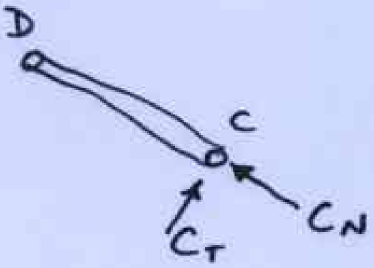
$$N_A = mg - N_B$$
$$= (200)(9.81) - 1690 = 272\ N$$

Max accln & N_A still > 0

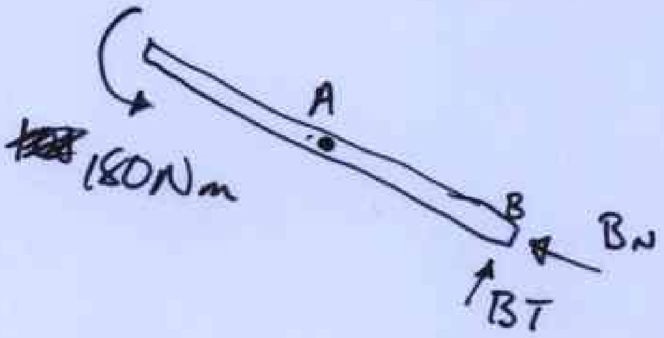
$\Rightarrow$ wheelie impossible

$\theta = 60^\circ$





Light Linkage $\Rightarrow C_T = 0$



$-|AB|\, B_t = +180\,\text{Nm}$

$\Sigma M = 0$

<u>Because link light</u>

$$B_t = \frac{180}{|AB|} = \frac{180}{0.6} = 300\,\text{N}$$