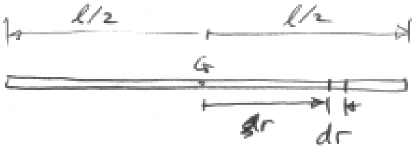
EN 2001 2nd Yr Applied Dynamics

DR. M. Conry Q.4

Show $I_G = \frac{1}{12} m l^2$ for bar of length l, mass m

$$I_o = \int r_o^2 \, dm$$



for rod, about G

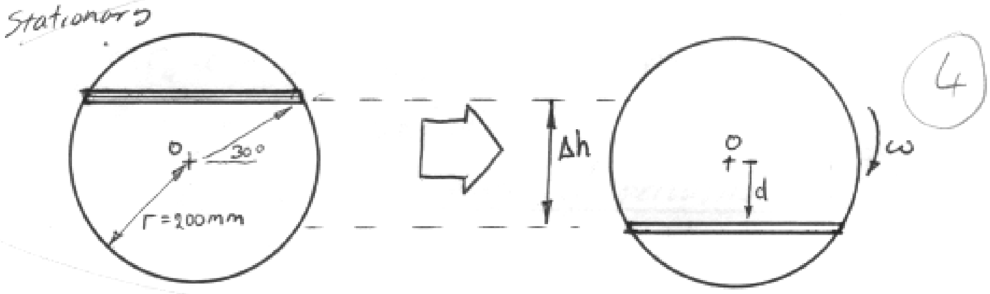
$$I_G = \int_{-\frac{l}{2}}^{+\frac{l}{2}} r^2 \, dm$$

$dm = \frac{m}{l} dr$ if cross section is uniform

So $I_G = \frac{m}{l} \int_{-\frac{l}{2}}^{+\frac{l}{2}} r^2 dr$ 16

$$= \frac{m}{l}\left[\frac{r^3}{3}\right]_{-l/2}^{+l/2} = \frac{m}{l}\left(\frac{l^3}{24} + \frac{l^3}{24}\right) = \underline{\frac{1}{12} m l^2}$$

Sketch Disk Before & after rotation



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Clearly, rod changes potential energy, Disk does not.

Rod = 2kg = m

Disk = 6kg = 3m

Smooth bearing at O. Conservation of Energy.

$$\cancel{T_1}^{\,0} + V_{1-2} = T_2$$

U_{1-2} ; $\Delta h = (2)(r)(\sin 30°) = r$

$\therefore V_{1-2} = mg\Delta h = mgr$ 10

$T_2 = \frac{1}{2} I_o \omega^2$

$I_o = I_{o_{DISK}} + I_{o_{ROD}}$. $\quad I_{o_{DISK}} = \frac{1}{2} m_{Disk} r^2 = \frac{3r^2}{2} m$

$I_{o_{ROD}} = I_{G_{ROD}} + m_{rod} d^2$ $\quad \left(d = r \sin 30° = \frac{r}{2}\right)$

10

$= \frac{1}{12} m l^2 + \frac{m r^2}{4}$ $\quad l = 2r\cos 30° = 2r\frac{\sqrt{3}}{2} = r\sqrt{3}$

$= \frac{m 3r^2}{12} + \frac{m r^2}{4}$

$I_{o_{ROD}} = \frac{1}{2} m r^2$

$\therefore I_o = m\frac{3r^2}{2} + \frac{1}{2} m r^2 = 2 m r^2$

$V_{1-2} = \frac{1}{2} I_o \omega^2 \Rightarrow \omega^2 = \frac{2V_{1-2}}{I_o} = \frac{\cancel{m} g r \cancel{2}}{\cancel{2} \cancel{m} r^2} = \frac{g}{r}$ 6

$$\boxed{\omega = \sqrt{g/r} = \sqrt{9.81/0.2} = 7.00 \text{ rads}^{-1}}$$

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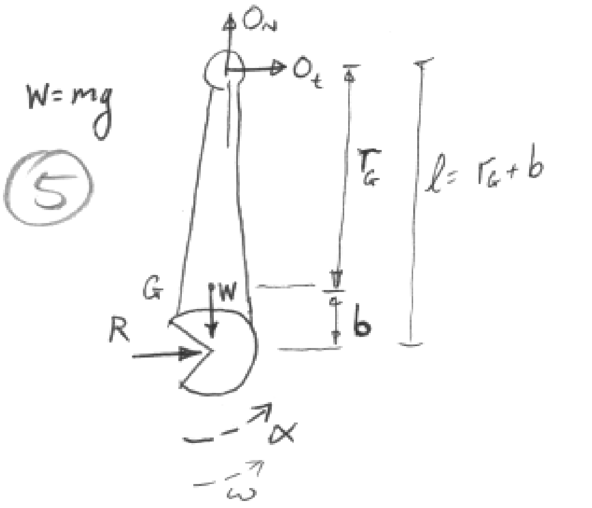
(a) at impact specimen applies force $\vec{R}$,

DRAW F.B.D



$$\sum F_t = m a_{G_t}$$

$$O_t + R = m a_{G_t} = m r_G \alpha \quad 4$$

$$\boxed{R = m r_G \alpha - O_t} \quad ①$$

4

$$\sum M_o = I_o \alpha$$

$$\Rightarrow \boxed{(R)(l) = I_o \alpha} \quad ② \qquad I_o = m k_o^2$$

Substitute ① into ②

$$4 \quad (m r_G \alpha - O_t)(l) = I_o \alpha = m k_o^2 \alpha \qquad \text{want minimum } \|O_t\| \text{; set to zero}$$

$$\cancel{m} r_G \cancel{\alpha}\, l = \cancel{m} k_o^2 \cancel{\alpha}$$

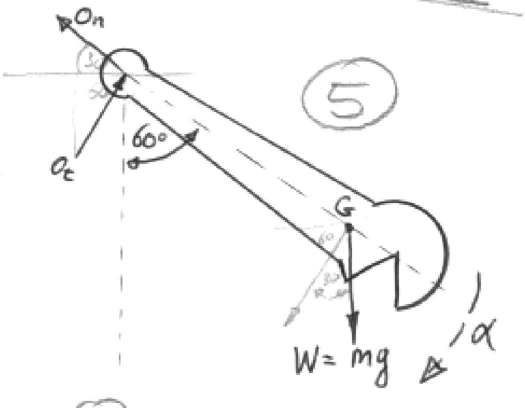
$$l = \frac{k_o^2}{r_G} \qquad \underline{\text{centre of percussion}}.. \quad 1$$

~~18~~ 20

$$\text{So} \quad l = \frac{(0.620)^2}{(0.6)} = 0.6407 \text{ m} \Rightarrow b = (l - 0.6) = 40.7 \text{ mm}$$

2

(b) at moment of release draw F.B.D.



$$\sum F_n = m a_{G_n}$$

$$O_n - mg\cos(60°) = m r \omega^2 = 0 \quad (\omega = 0) \quad 4$$

$$O_n = (34)(9.81)(0.5) = \underline{\underline{166.77\text{N}}}$$

$$\sum M_o = I_o \alpha$$

$$mg \sin(60°) r_G = m k_o^2 \alpha$$

$$\Rightarrow \alpha = \frac{(34)(9.81)(0.866)(0.6)}{(34)(0.620)^2} = 13.26 \text{ rads}^{-2} \quad 4$$

$$\sum F_t = m a_{G_t}$$

$$O_t - mg\sin(60°) = -m r_G \alpha \quad 4$$

$$O_t = (34)(9.81)(0.866) - (34)(0.6)(13.26) = \underline{\underline{18.35\text{N}}}$$

~~22~~ 20

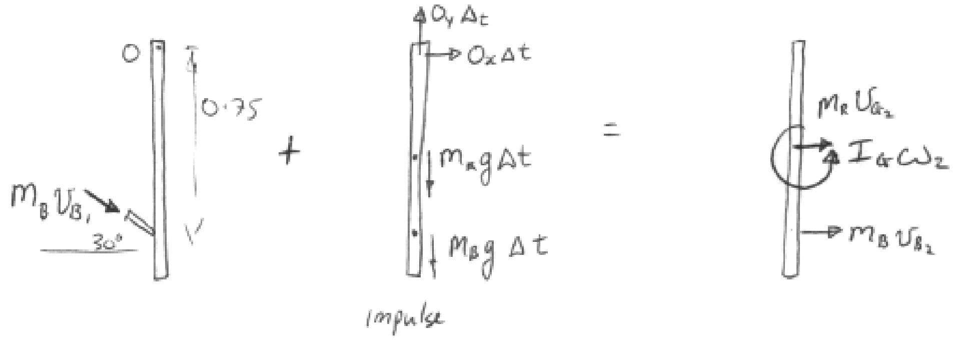
$$\|O\| = \sqrt{(18.35)^2 + (166.77)^2} = \boxed{167.8\text{N}} \quad \underline{\text{ANS}} \quad 3$$

EN 2007   2nd Yr applied Dynamics

Dr. M. Conry   Q.3

Look at bullet plus rod as a single system

impulse - momentum



(6)

(24)

no angular impulse about O

$\Rightarrow$ angular momentum <u>about O</u> conserved

$$\Sigma(H_o)_1 = \Sigma(H_o)_2$$

(6)

① $m_B v_{B_1} \cos(30°)(0.75) = m_B v_{B_2}(0.75) + m_R v_{G_2}(0.5) + I_G \omega_2$

from kinematics $\begin{cases} v_{B_2} = 0.75\,\omega_2 \\ v_{G_2} = 0.50\,\omega_2 \end{cases}$ (6)

also $I_G = \frac{1}{12} m_R l^2 = \frac{m_R}{12}(1.0)^2$

Substitute into ① & get $\omega_2 = 0.623$ rad s^{-1} (6)

can approximate well by ignoring mass of bullet on right hand side

also can use I_o

$$(m_B)(v_{B_1})(\cos 30°)(0.75) \approx I_o \omega_2 = \frac{1}{3} m l^2 \omega_2 \Rightarrow \omega_2 \approx 0.624 \text{ rad s}^{-1}$$

Energy: <u>initial</u> = $\frac{1}{2} m_B v_B^2 = (0.5)(0.004)(400)^2 = 320$ J   (10)

<u>after</u> : rod : $\frac{1}{2} I_o \omega_2^2 = (0.5)(\frac{1}{3})(5)(1.0)^2(0.624)^2 = 0.324$ J

bullet = $\frac{1}{2} m_B v_{B_2}^2 = (0.5)(0.004)(0.75)^2(0.624)^2 = 0.000438$ J
NEGLIGIBLE

"Lost" kinetic energy will have been dissipated in system as bullet embeds in rod. Also, support/pivot will have had to hold rod vertically as vertical component was spent   (6)

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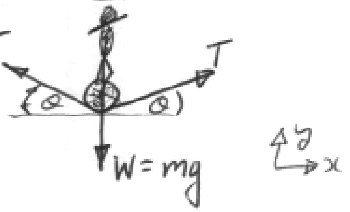
Draw F.B.D. for acrobat + unicycle

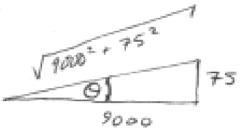


$$\theta = \tan^{-1}\left(\frac{75}{9000}\right) = 0.477^\circ$$ 5 (v. small angle).

$$W = (50)(9.81) = 490.5\,N$$ 5

$$\Sigma F_y = 0 \quad (2)(T)(\sin\theta) = 490.5$$ 7

$$T = \frac{490.5}{(2)(\sin\theta)} = \frac{490.5}{(2)\frac{75}{\sqrt{9000^2+75^2}}}$$



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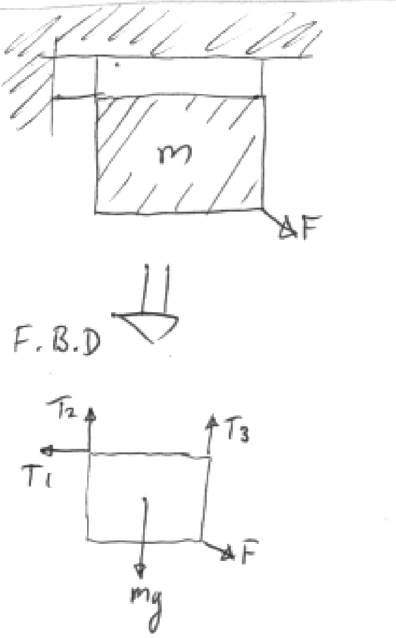
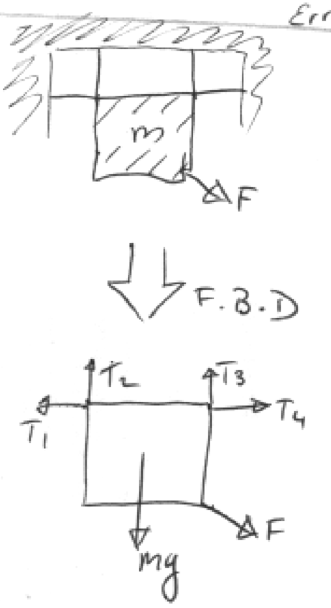
$$T = 29430\,N = 29.43\,kN$$

NOTE... it is an acceptable approximation to say $\sin\theta \approx \frac{75}{9000}$ 1
Error is v. small.





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3 unknowns .. T_1, T_2, T_3
3 equations $\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_o = 0$

⇒ Soluable
"Statically Determinate"

now 4 unknowns T_1, T_2, T_3, T_4

still only 3 equations
$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_o = 0$

⇒ insoluble
"statically indeterminate"
redundant constraint present and need more info to solve
e.g. look @ deformation of supports.