1 3rd Year Design and Production

Fatigue – Lecture 5

2 Stress Concentration

2.1 Applied to Goodman Criteria

- Nominal Mean Stress Method (ductile material)

  Apply K_f only to alternating component

  $$\frac{[K_f S_a]}{S_n} + \frac{S_m}{S_u} = \frac{1}{FS}$$

- Residual Stress Method (Always use for brittle material. For ductile materials, adjust for yielding and resultant residual stress if predicted stress $> S_y$)

  Apply K_f to alternating **and** mean components

  $$\frac{[K_f S_a]}{S_n} + \frac{[K_f S_m]}{S_u} = \frac{1}{FS}$$

Different texts will make different recommendations on this.

3 Equivalent Stress Equations

To account for situation where there is a combination of bending, shear, and/or axial stresses it is necessary to determine the equivalent stress that is created. Different forms are possible…

- Maximum Shear Stress

  $$\tau_{\max} = \sqrt{\left(\frac{\sigma_{\text{eq}}}{2}\right)^2 + \tau_{\text{eq}}^2}$$

- Maximum Normal/Principle Stress

  $$\sigma_{\max} = \frac{\sigma_{\text{eq}}}{2} + \sqrt{\left(\frac{\sigma_{\text{eq}}}{2}\right)^2 + \tau_{\text{eq}}^2}$$

- Von-Mises / Distortion Energy Theory

  $$\sigma_{\max} = \sqrt{\sigma_{\text{eq}}^2 + 3\tau_{\text{eq}}^2}$$

4 Equivalent Stress Equations

4.1 How to Use these Relations

Juvinall recommends the following policy:

- Find the **equivalent alternating bending stress** using distortion energy theory as:

$$\sigma_{\text{ea}} = \sqrt{\sigma_a^2 + 3\tau_{\text{a}}^2}$$

- Find the **equivalent mean bending stress** as the maximum principle stress:

$$\sigma_{\text{em}} = \frac{\sigma_m}{2} + \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2}$$

Shigley recommends the use of the distortion energy equation to find both alternating and mean stresses.

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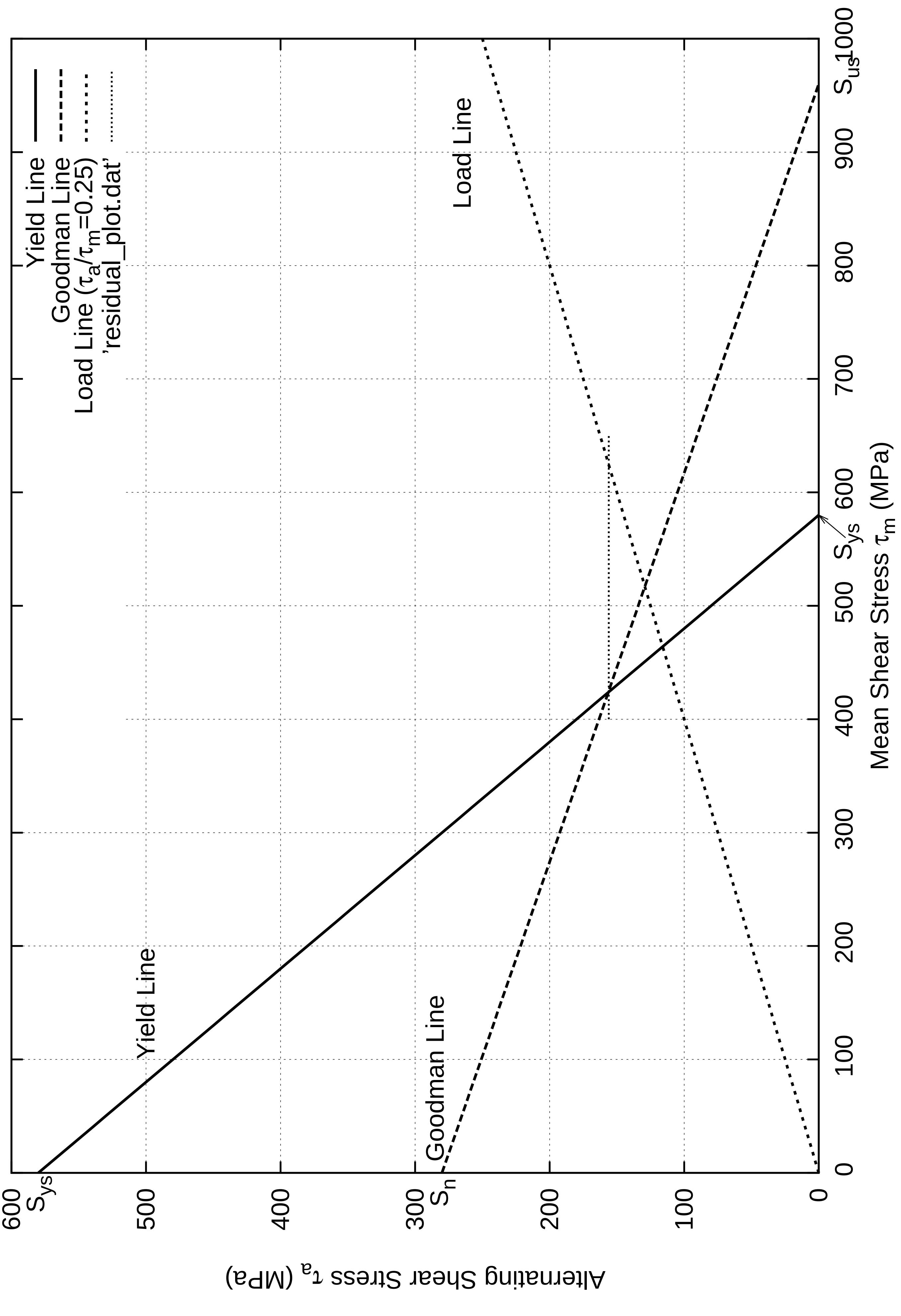
5 Cumulative Fatigue Damage

We have studied varying loads. However, we have assumed that σ_m and σ_a have not varied over time. Often this is not the case.

5.1 Palmgren/Miner Rule

- If $n_1, n_2, n_3, n_4, \ldots, n_k$, are the number of cycles accumulated at specific stress levels
- And $N_1, N_2, N_3, N_4, \ldots, N_k$, are the lifetimes predicted at these stress levels
- Then failure will occur when

$$\sum_{j=1}^{j=k} \frac{n_j}{N_j} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} + \cdots + \frac{n_k}{N_k} = 1$$


Yield Line
Goodman Line
Load Line (τ_a/τ_m=0.25)
'residual_plot.dat'
Load Line
Yield Line
Goodman Line
Mean Shear Stress τ_m (MPa)
Alternating Shear Stress τ_a (MPa)
S_{ys}
S_{us}
S_n
0
100
200
300
400
500
600
700
800
900
1000