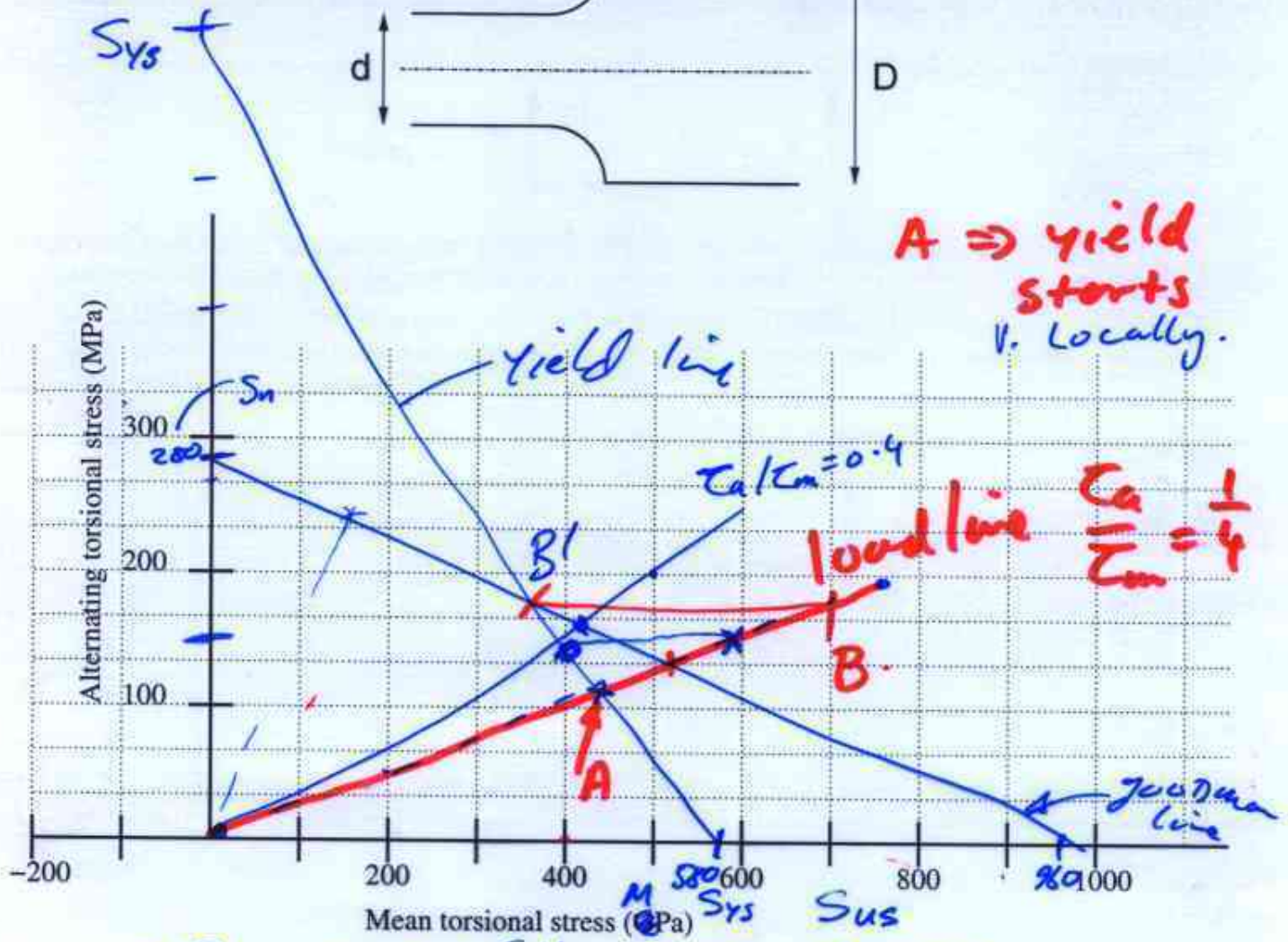
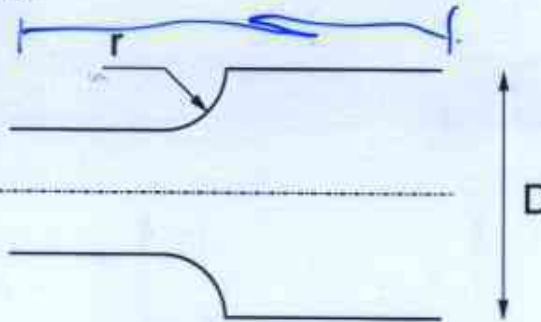


9th-January-2004

**Problem** A shaft must transmit a torque of 1000 Nm, with superimposed torsional vibrations causing an alternating torque of 250 Nm. A safety factor of 2 should be applied to both loads.

A heat treated alloy steel is to be used, with  $S_u = 1.2\text{GPa}$  and  $S_y = 1.0\text{GPa}$ . The shaft will have a shoulder, with  $D/d=1.2$  and  $r/d=0.05$  as show in the diagram. A good quality commercial ground finish is to be specified, what diameter is required for infinite life?



$S_n = ? \quad S_n' \approx \frac{S_u}{2} = \frac{1200}{2} = 600 \text{ MPa}$

$S_n = C_L C_D C_S S_n'$   
 Torsion  $\Rightarrow C_L = 0.58$ ;  $C_S = 0.9$  (ground).

$C_D$  depends on  $d$  assume  $10 < \phi < 50 \text{ mm}$   
 $C_D = 0.9$  (for torsion).

$S_n = (600)(0.58)(0.9)(0.9) \approx 280 \text{ MPa}$

$S_{us} = (0.8)(S_u) = 960 \text{ MPa}$      $S_{ys} \approx (0.58) S_y = 580 \text{ MPa}$  MConry

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$$\tau_m = ? \quad \tau_a = ?$$

$$\tau_m = \left( \frac{16 T_m}{\pi d^3} \right) k_f$$

$$\tau_a = \left( \frac{16 T_a}{\pi d^3} \right) k_f$$

need  $k_f \rightarrow$  need  $k_t \approx q$

$$k_t \quad r/d = 0.05 \quad D/d = 1.2$$

$$k_t = 1.57 \text{ from Handout.}$$

$q$  depends on  $r$ , which we don't know

$\Rightarrow$  be conservative

just use  $S_u$  to get  $q$

$$\Rightarrow q = 0.95$$

$$k_f = 1 + (1.57 - 1)(0.95) \approx 1.54$$

subst into  $\tau_m \quad \tau_a$

$$\tau_m = \overset{\text{F.S.}}{2} \left[ \frac{16 (1000)}{\pi d^3} \right] \overset{k_f}{1.54} = 15685/d^3$$

$$\tau_a = 2 \left[ \frac{16 (250)}{\pi d^3} \right] \overset{k_f}{1.54} = \frac{3922}{d^3}$$

$$\left[ \begin{array}{l} \tau_a = 0.25 \text{ approx.} \\ \tau_m \end{array} \right. \rightarrow \text{load line}$$

if you apply  $K_t$  only to  
 ~~$\tau_a$~~   $\tau_a$

$$\tau_a = \frac{3922}{d^3} \quad \tau_m = \frac{15685}{1.54 d^3}$$

$$\approx \frac{10000}{d^3}$$

$$\frac{\tau_a}{\tau_m} = 0.4 \quad (\text{not } 0.25)$$

this load line has also been plotted on the constant Life fatigue diagram, and it will give us a somewhat different failure point

either way, find  $\tau_a$  at failure (approx 150 MPa)

Substitute into

$$\tau_a = \frac{3922}{d^3} \Rightarrow d^3 = \frac{3922}{\tau_a} \quad \text{need to be careful with units.}$$

$\tau_a$  was calculated in terms of Nm, so stick with that

$$\tau_a = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

$$d = \sqrt[3]{\frac{3922}{150 \times 10^6}} = 0.0296 \approx 30 \text{ mm}$$

