
1 3rd Year Design and Production

Fatigue – Lecture 3

2 Fatigue

2.1 Revision

Fatigue strength as obtained from R. R. Moore fatigue tests can be used to predict the fatigue strength of other components. We use modifying factors:

$$S_n = S'_n C_L C_G C_S$$

- S_n is the actual (calculated) fatigue strength
- S'_n is data from SN curve (experiment or estimate)
- C_L modifying factor for **loading type**
- C_D (a.k.a. C_G) modifying factor for **gradient/size**
- C_S modifying factor for **surface finish**

In general $S_n < S'_n$

3 Endurance Limit Factors

	Bending	Axial	Torsion
(a) Endurance Limit $S_n = S'_n C_L C_G C_S$			
C_L – Load Factor	1	1	0.58
C_S – Surface Factor	chart	chart	chart
C_D – Dimension Factor ($0 < \varnothing < 10\text{mm}$)	1.0	0.7 to 0.9	1.0
C_D – Dimension Factor ($10\text{mm} < \varnothing < 50\text{mm}$)	0.9	0.7 to 0.9	0.9
(b) 10^3 Cycle Strength	$0.9 S_u$	$0.75 S_u$	$0.9 S_{us}$

$$S'_n \approx 0.5 S_u$$

$$S_u \approx 0.5(Bhn)[ksi]$$

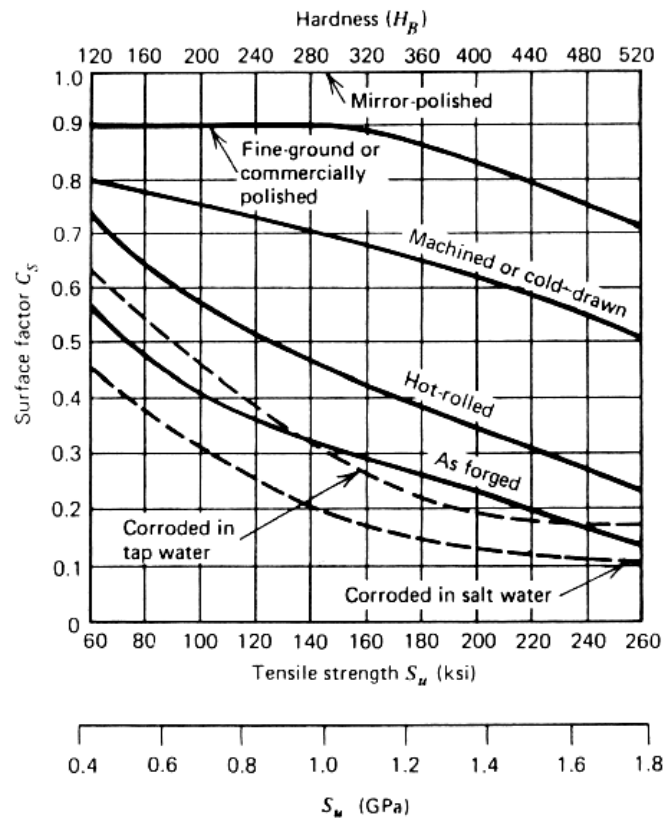
$$S_{us} \approx 0.8 S_u \text{ for steel}$$

$$S_{ys} \approx 0.58 S_y \text{ Max distortion energy theory}$$

$$S_{ys} \approx 0.50 S_y \text{ Max shear stress theory}$$

4 Endurance Limit Factors

4.1 Surface Finish

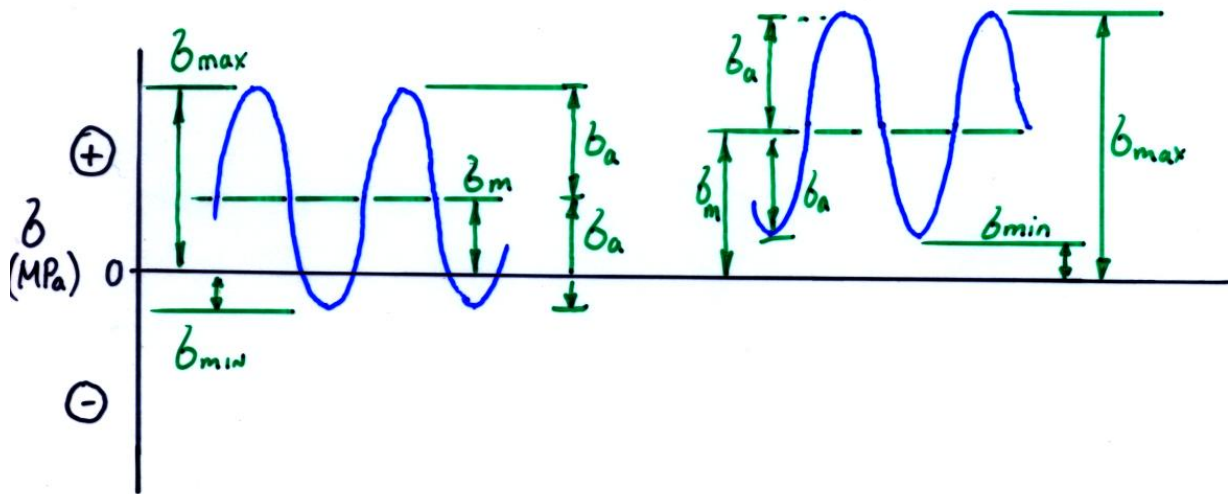


5 Mean and Alternating Stress

- We have been discussing **Fully Reversed Stress States** \Rightarrow mean stress is zero.
- In many real situations, this is not the case.
- Some notation:
 - ◇ σ_m mean stress
 - ◇ σ_{max} maximum stress
 - ◇ σ_{min} minimum stress
 - ◇ σ_a amplitude of fluctuating component
- A Mathematical example:
 - ◇ $\sigma = \sigma_m + \sigma_a \sin(\omega t)$
 - ◇ $\Rightarrow \sigma_{max} = \sigma_m + \sigma_a$ and $\sigma_{min} = \sigma_m - \sigma_a$
 - ◇ $\Rightarrow \sigma_m = 0.5(\sigma_{max} + \sigma_{min})$

6 Mean and Fluctuating Stress

MEAN & ALTERNATING STRESS

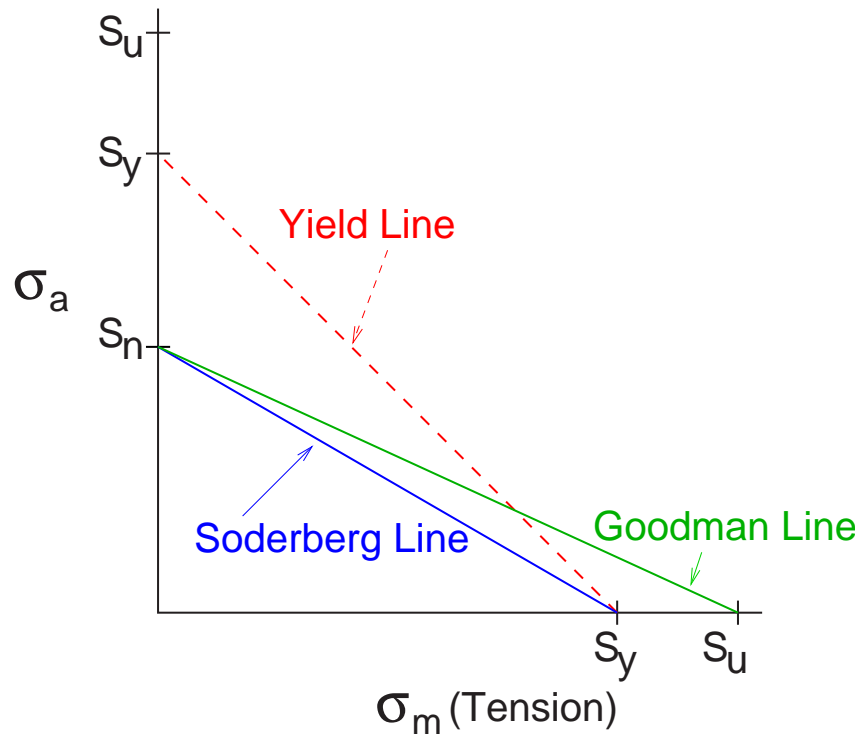


- Presence of a Static **Tensile** Stress reduces the amplitude of Reversed Stress that can be applied
 - Presence of a Static **Compressive** Stress slightly increases amplitude of Reversed Stress that can be applied (effect usually ignored)
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7 Constant Life Fatigue Diagrams

- To look at effect/interaction of σ_m and σ_a , we aim to draw a constant life fatigue curve.
 - x-axis: mean stress σ_m
 - y-axis: fluctuating amplitude σ_a
 - If $\sigma_a = 0$, **Static Failure** (intersection of curve with x-axis)
 - May use S_y or S_u depending on application
 - If $\sigma_m = 0$, **Totally Reversed Load** (intersection of curve with y-axis)
 - Data (maybe modified) from SN diagram: S_n (for infinite life)
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8 Constant Life Fatigue Diagrams



9 Constant Life Fatigue Diagrams

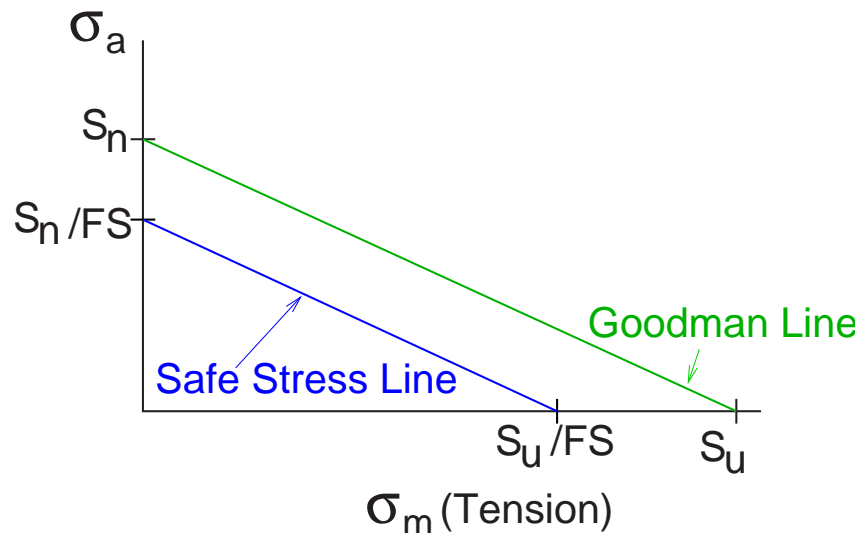
9.1 Equations

- Soderberg Line is based on S_y and S_n
→ $S_a/S_n + S_m/S_y = 1$
- Goodman Line is based on S_u and S_n
→ $S_a/S_n + S_m/S_u = 1$
- Gerber Locus is based on S_u and S_n
→ $S_a/S_n + (S_m/S_u)^2 = 1$
→ Parabolic
- Distortion Energy Locus is based on S_u and S_n
→ $(S_a/S_n)^2 + (S_m/S_u)^2 = 1$
→ Elliptical

10 Factor of Safety

- Illustrated here for Goodman Criteria (can use Soderberg, etc., also).
- FS is the factor of safety
- Safe stress line is parallel to original Goodman line

$$\frac{S_a}{S_n} + \frac{S_m}{S_u} = \frac{1}{FS}$$



11 Summary of CLF Diagrams

- Four lines relevant for infinite life:
 - Soderberg, Goodman, Yield-line, Load-line
- Two additional failure loci (less conservative)
 - Gerber (parabolic), Distortion-Energy (elliptical)
- Which failure criteria to use?:
 - If no yielding allowed, Soderberg will be conservative
 - If we don't know S_u , then use Soderberg
 - If S_u is known, and we want to find infinite life, with yielding permitted, then use Goodman line
 - If S_u is known, and we want to find infinite life, with yield onset permitted, then use Goodman line and yield line to define the safe zone (Modified Goodman Criterion).

Two acetates

12 Stress Concentration

- Surface features and flaws can lead to locally elevated stresses
- Tables/charts allow us to look up K_t
 - Theoretical or Geometric stress concentration factors
- Materials have different **notch sensitivities**: q
 - Material with a lot of flaws not damaged much by one more
⇒ low notch sensitivity

- Very perfect material is significantly damaged by addition of a notch
⇒ high notch sensitivity

- Calculate **Fatigue Stress Concentration Factor** K_f using K_t and q :

$$K_f = 1 + q(K_t - 1)$$

13 Stress Concentration

What do we apply K_f to?

- Ductile Materials
 - Nominal Mean Stress Method
 - Apply K_f only to alternating stress
- Brittle Materials
 - Residual Stress Method
 - Apply K_f to both alternating stress and mean stress