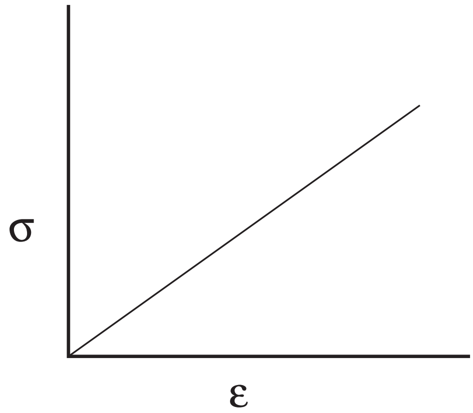
1 3rd Year Engineering Materials

Viscoelasticity – Lecture 1

2 Elastic Behaviour



$\sigma = E\epsilon$

- Apply a load and the deformation is directly proportional to the load.
- Remove the load and the deformation returns to zero
- Time duration is very small (approximately zero for many applications)

3 Viscous Behaviour

$$\tau = \nu \frac{\mathrm{d}\gamma}{\mathrm{d}t}$$

- τ is the shear stress
- γ is the shear strain
- ν is the viscosity.
- Stress is proportional to the **rate of strain**
- Equally, rate of strain (as opposed to strain itself) is proportional to stress
- If we have a constant stress, then strain will grow forever

Extreme Viscous behaviour is observed when you try to stir treacle, or tar.

4 Viscoelastic Behaviour – Creep

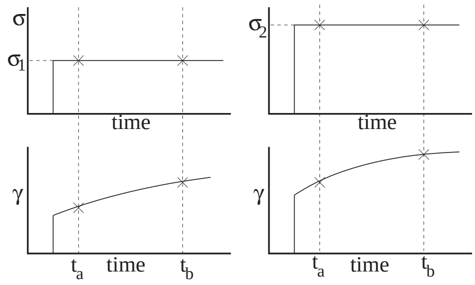
Mixture of Elasticity and Viscosity...
Polymeric solids are viscoelastic

- Apply a constant force to a polymer filament:
  - There is an immediate elastic-type strain.
  - If the load remains in place, strain increases slowly with time due to molecular realignment in the solid induced by the stress.
  - A back-stress builds up in the solid due to this deformation, which eventually halts the strain.

- Remove the load:
  - Initial elastic recovery (short time-scale)
  - Molecules continue to slowly recover former orientation (under influence of back-stress)
  - Strain returns to zero

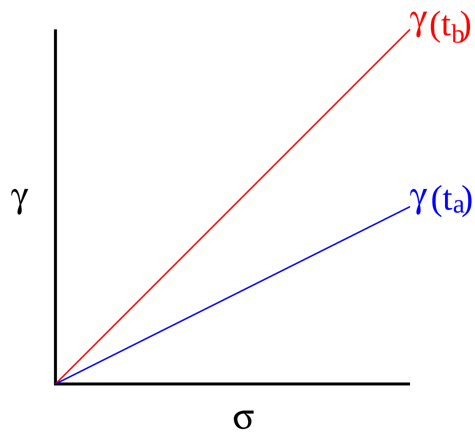
5 Viscoelastic Behaviour – Creep

5.1 Some Pointers



- Must consider stress and time for which it is applied
- Temperature changes the creep behaviour
- Polymers do not creep indefinitely
- Polymers recover completely once the stress is removed

6 Linear Viscoelasticity



- Creep is linear viscoelastic at low stresses
  - i.e. for any time t, σ is a linear function of γ
  - Applies for $\epsilon < 0.005$

7 Viscoelasticity

7.1 Isochronal

An isochronal is a plot of stress versus strain for a given time.

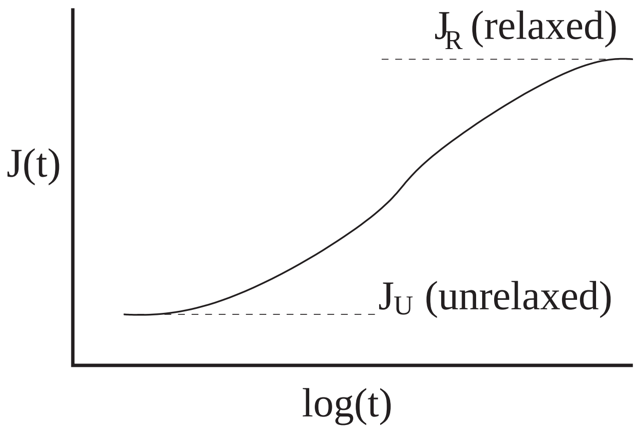
7.2 Comparisons with Metals

- In metals creep is not linear viscoelastic
- In metals creep is not recoverable
- In metals creep is only significant at high temperatures (i.e. above half the melt temperature)
  - In polymers it occurs at all temperatures above $-200^\circ\ C$

8 Creep Compliance

Compliance is simply the inverse/reciprocal of stiffness

$$J(t) = \frac{\gamma(t)}{\sigma}$$



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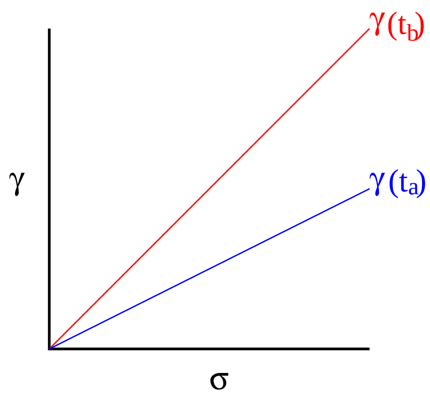
9 Creep Compliance

- For a viscoelastic material, this varies with time
- For a **linear viscoelastic** material, this does **not** vary with stress

Essentially, we're looking at the slopes of the isochronals. If they are straight lines, then the slope is constant with respect to stress. Different isochronals have different slopes though, so $J(t)$ does vary with t.



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10 Example

A spherical pressure vessel is moulded from a polymer whose one month tensile creep compliance $J(1\text{ month})$ is 2GPa^{-1}. The diameter of the vessel is 400mm and the wall thickness is 5mm. Poisson's ratio for the polymer is $\nu = 0.41$ and is constant.

Internal pressure is constant, and produces a tensile stress of 1.6 MPa acting uniformly in the plane of the vessel wall

Find, at the end of the month

- The change in Diameter
- The change in Wall Thickness

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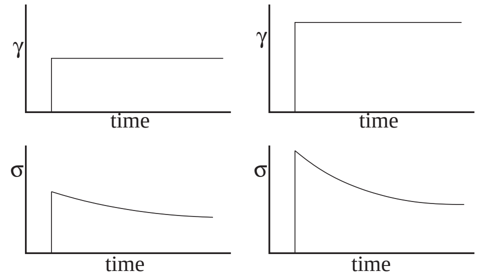
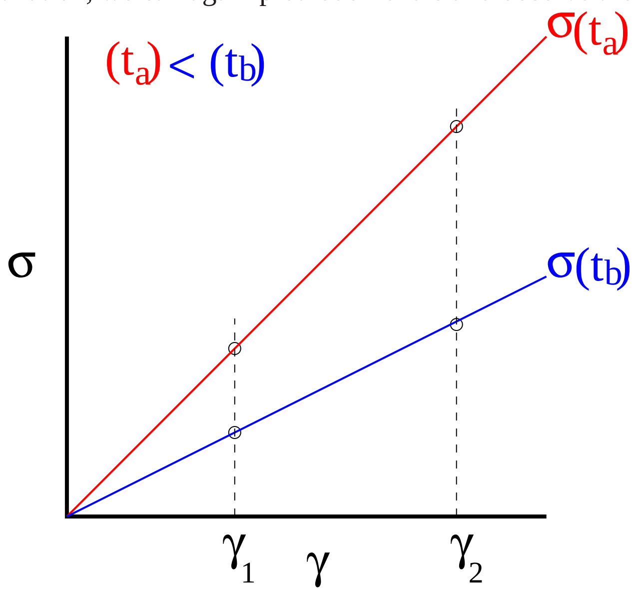
11 Stress Relaxation



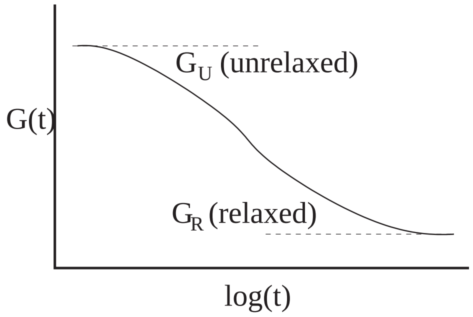
- Stress Relaxation is the flip-side of creep
- If strain is kept constant, then the stress will gradually reduce

12 Stress Relaxation

Based on data from stress relaxation, we can again plot isochronals and observe the linear viscoelastic behaviour. . .



13 Stress Relaxation Modulus



Closely related to the creep compliance

$$G(t) = \frac{\sigma(t)}{\gamma}$$

Based on data from stress relaxation, we can again plot isochronals and observe the linear viscoelastic behaviour. . .