2D problem ... So:

$\therefore\ \vec{r}_i = x_i\hat{\imath} + y_i\hat{\jmath} \qquad \vec{a}_P = a_{P_x}\hat{\imath} + a_{P_y}\hat{\jmath}$

$\vec{\alpha} = \alpha\hat{k}$ and $\vec{M}_{P_i} = M_{P_i}\hat{k}$

So $$M_{P_i}\hat{k} = m_i\Big\{(x_i\hat{\imath} + y_i\hat{\jmath})\times[a_{P_x}\hat{\imath} + a_{P_y}\hat{\jmath}] + (x_i\hat{\imath}+y_i\hat{\jmath})\times[\alpha\hat{k}\times(x_i\hat{\imath}+y_i\hat{\jmath})]\Big\}$$

$$M_{P_i}\hat{k} = m_i\Big\{-y_i a_{P_x} + x_i a_{P_y} + \alpha\underbrace{(x_i^2+y_i^2)}_{\text{note } x^2+y^2=r^2}\Big\}\hat{k}$$

if we let $m_i \to dm$ & integrate over mass m we get ...

$$\underbrace{\sum M_P}_{\text{moment of external forces about point P (internal moments cancel out)}} = -\underbrace{\left(\int_m y\,dm\right)}_{m y_G} a_{P_x} + \underbrace{\left(\int_m x\,dm\right)}_{m x_G} a_{P_y} + \underbrace{\left(\int_m r^2\,dm\right)}_{\text{M.O.Inertia about P } I_P}\alpha$$

i.e. the integrals locate centre of mass

$$\boxed{\sum M_P = m\vec{\rho}\times\vec{a}_P + I_P\alpha} \qquad \boxed{\vec{\rho} = x_G\hat{\imath} + y_G\hat{\jmath}}$$

Say $P = G$ (center of mass)

$$\Rightarrow \underline{\sum M_G = m\vec{0}\times\vec{a}_G + I_G\alpha = I_G\alpha}$$

OR Say P has no acc'n

$$\Rightarrow \sum M_P = m\vec{\rho}\times\vec{0} + I_P\alpha$$

$$\underline{\sum M_P = I_P\alpha}$$