

Another form is:

$$\sum \vec{M}_P = I_P \alpha + \vec{r} \times m \vec{a}_P$$

Here P is a point fixed to the body

\vec{a}_P is accel of P

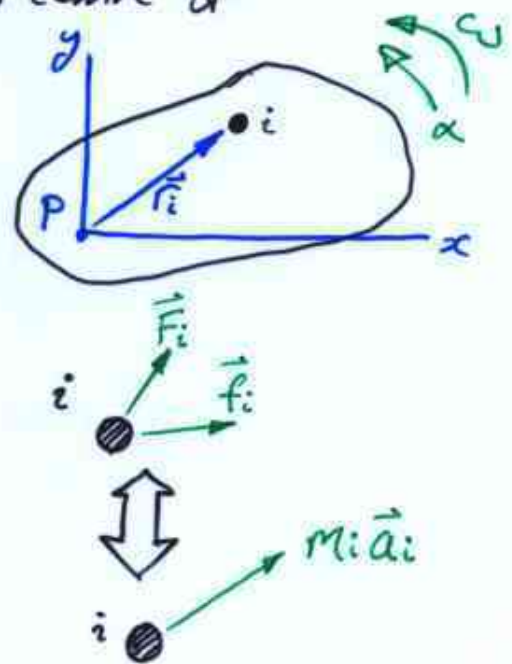
\vec{r} is vector from P to mass centre G

Consider small particle i

\vec{F}_i = Resultant External force acting on particle

\vec{f}_i = resultant internal force due to interactions with adjacent particles

mass is m_i



Sum moments about P :

$$\vec{r}_i \times \vec{F}_i + \vec{r}_i \times \vec{f}_i = \vec{r}_i \times m_i \vec{a}_i$$

$$\text{i.e. } \vec{M}_{P_i} = \vec{r}_i \times m_i \vec{a}_i$$

in general, Point P can have an acceleration

$$\Rightarrow \vec{a}_i = \vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i$$

$$\text{so } \vec{M}_{P_i} = m_i \vec{r}_i \times (\vec{a}_P + \vec{\alpha} \times \vec{r}_i - \omega^2 \vec{r}_i)$$

$$= m_i [\vec{r}_i \times \vec{a}_P + \vec{r}_i \times (\vec{\alpha} \times \vec{r}_i) - 0]$$

$$\boxed{\vec{r}_i \times \vec{r}_i = 0}$$

