

EXAMPLES: FIND MOI of a SOLID Sphere
about a diameter

FIND I_{zz}

USE DISK AS ELEMENT

$$\begin{aligned} dI_{zz} &= \frac{1}{2} (dm) r^2 \\ &= \frac{\pi \rho}{2} (R^2 - z^2)^2 dz \end{aligned}$$

$$I_{zz} = \int dI_{zz}$$

$$= \int_{-R}^{+R} \frac{\pi \rho}{2} (R^2 - z^2)^2 dz$$

$$= \frac{\pi \rho}{2} \int_{-R}^{+R} (R^4 - 2R^2 z^2 + z^4) dz$$

$$= \frac{\pi \rho}{2} \left[R^4 z - \frac{2R^2 z^3}{3} + \frac{z^5}{5} \right]_{-R}^{+R}$$

$$= \frac{\pi \rho}{2} \left(\left[R^5 - \frac{2R^5}{3} + \frac{R^5}{5} \right] - \left[-R^5 + \frac{2R^5}{3} - \frac{R^5}{5} \right] \right)$$

$$= \frac{\pi \rho}{2} (2) \left(R^5 - \frac{2R^5}{3} + \frac{R^5}{5} \right)$$

$$= \frac{8\pi \rho R^5}{15}$$

NOTE $m = \rho \frac{4}{3} \pi r^3$ for sphere

$$\therefore I_{zz} = \frac{2}{5} m R^2$$

$$k = \sqrt{\frac{I}{m}} = \sqrt{\frac{2}{5}} R$$

radius of gyration

