Simplification: PLANE MOTION

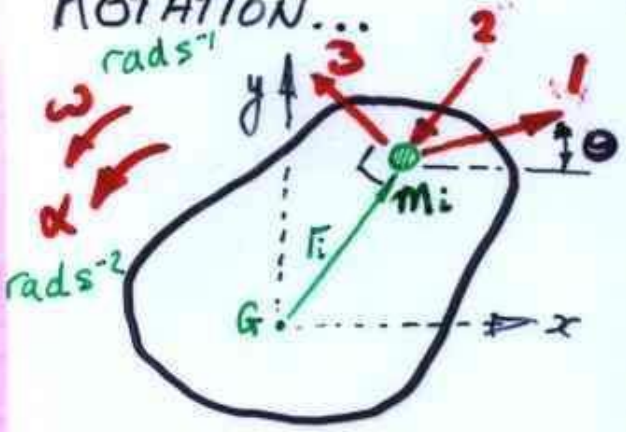
ALL FORCES, MOVEMENT (TRANSLATION & ROTATION) IN 1 PLANE.

ALL MOMENTS ⊥ to this PLANE

↳ CAN USE SCALAR NOTATION FOR MOMENT, ANGULAR ACCLN, ETC.,

STILL: $\boxed{\sum \vec{F} = m\vec{a}_G}$ ... TRANSLATION

ROTATION...



3 COMPONENTS OF ACCEL.
⇒ 3 COMP. of force

1. $m_i\vec{a}_G$
2. $m_i r_i \omega^2$ ... toward G
3. $m_i r_i \alpha$ ... ⊥ to $\vec{r}_i$

LOOK AT MOMENT OF EACH COMPONENT: **ABOUT G**

1. $M'_G = m_i a_G \sin(\theta)\, x_i - m_i a_G \cos(\theta)\, y_i$
2. $M^2_G = 0$ (DIRXN of ω DOES not matter)
3. $M^3_G = (m_i r_i \alpha \times r_i) = m_i r_i^2 \alpha$

ADD 1. 2. & 3. ... then sum all the m_i that make up THE BODY:

$$\sum M_G = a_G \sin(\theta) \underbrace{\sum m_i x_i}_{\text{ZERO}} - a_G \cos(\theta) \underbrace{\sum m_i y_i}_{\text{ZERO}} + \alpha \sum m_i r_i^2$$

BECAUSE ORIGIN @ G

$$\sum M_G = \alpha \sum m_i r_i^2 \Rightarrow \boxed{\sum M_G = \alpha \int r^2 dm = \alpha I_G}$$

(α: const over BODY)