Dr. Michael Conry

michael.conry@ucd.ie

Engineering Mechanics

J.L. Meriam,
L.G. Kraige,

Wiley

Vol 1
Statics

Vol 2
Dynamics

NB

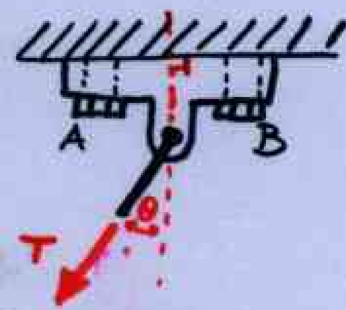
FORCE SYSTEMS:

DEFINE FORCE IN TERMS OF Acceleration

$N = kg\,m\,s^{-2}$ $\quad \vec{F} = m\vec{a}$ ... Newton's 2^{ND} LAW.

FORCE IS A VECTOR $\Rightarrow$ need DIRECTION AND MAGNITUDE TO fully specify any given $\vec{F}$

DISTINGUISH BETWEEN INTERNAL AND EXTERNAL FORCES:



EXTERNAL FORCES
- TENSION T IN Rope
- REACTION FORCES IN BOLTS A,B
- FORCE EXERTED BY WALL ON BRACKET

INTERNAL FORCES
- STRESSES WITHIN BRACKET MATERIAL

IN THIS COURSE WE WILL STUDY EXTERNAL forces.

IT WILL BE ASSUMED THAT THE BODIES BEING STUDIED ARE RIGID AND DO NOT DEFORM. It is IMPORTANT TO BE AWARE OF THIS ASSUMPTION AS IT DOES NOT ALWAYS apply

A SECOND CATEGORIZATION:

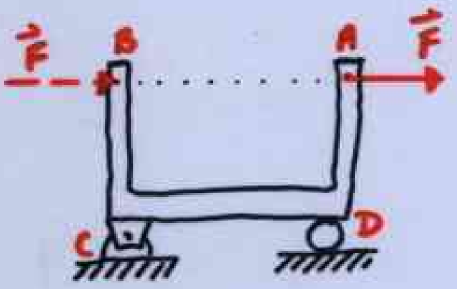
CONTACT FORCES: DUE TO DIRECT PHYSICAL CONTACT BETWEEN BODIES

BODY FORCES: e.g. gRAVITY, magnetic force, BUOYANCY.

Some basic principals:

THESE APPLY WHERE RIGID BODIES ARE STUDIED (i.e. THROUGHOUT THIS COURSE)

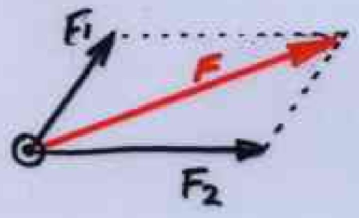
TRANSMISSIBILITY:

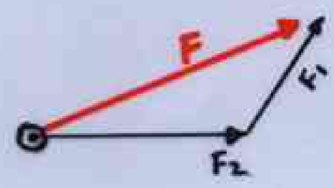


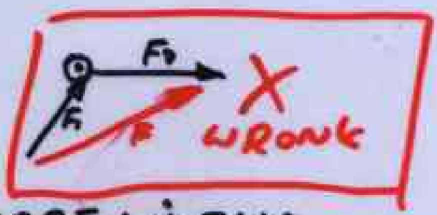
A FORCE CAN BE MOVED ALONG ITS LINE OF ACTION WITHOUT CHANGING ITS NET EXTERNAL EFFECT. i.e. WHETHER $\vec{F}$ is AT POINT A OR B REACTIONS AT C & D the SAME.
"SLIDING VECTOR"

VECTOR ADDITION:

CONCURRENT FORCES $\vec{F}_1, \vec{F}_2$ are added using PARALLELOGRAM LAW TO GIVE RESULTANT $\vec{F}$



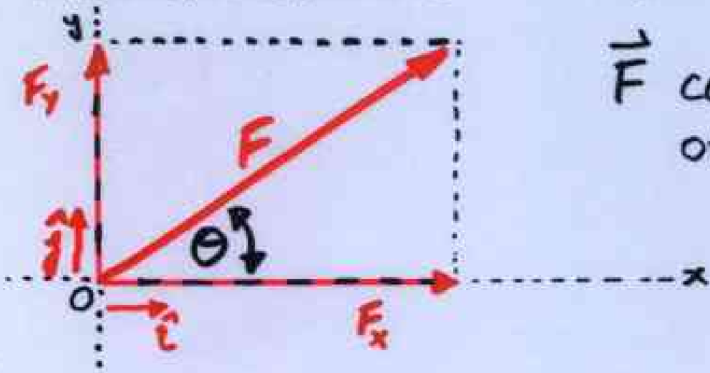




TRIANGLE LAW ALSO USED BUT MORE LIKELY TO GIVE ERRORS

THE INVERSE OF THIS IS THAT A VECTOR CAN BE EXPRESSED AS A SUM OF 2 OR MORE COMPONENT VECTORS. OFTEN WE CHOOSE THESE AT RIGHT ANGLES, THOUGH OTHER CHOICES ARE SOMETIMES MORE CONVENIENT

RECTANGULAR COMPONENTS ... 2D



$\vec{F}$ can be given as the sum of components $\vec{F_x}$ and $\vec{F_y}$

(USE BROKEN LINES FOR COMPONENTS)

UNIT VECTORS $\hat{i}$ & $\hat{j}$ ARE USEFUL TO INTRODUCE

then $\vec{F_x} = \|F_x\| \hat{i}$; $\vec{F_y} = \|F_y\| \hat{j}$ AND

$$\vec{F} = \|F_x\| \hat{i} + \|F_y\| \hat{j}$$

LOOKING AT TRIGONOMETRY WE SEE

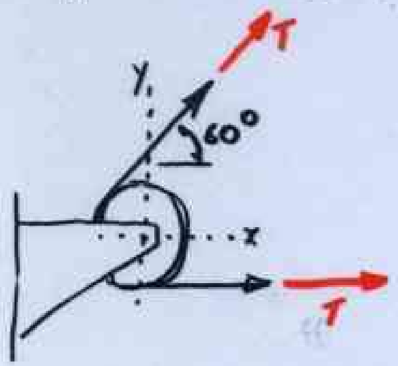
$$\|F_x\| = \|F\| \cos(\theta) \qquad \|F_y\| = \|F\| \sin(\theta)$$

$$\|F\| = \sqrt{\|F_x\|^2 + \|F_y\|^2} \qquad \tan(\theta) = \|F_y\| / \|F_x\|$$

WHERE you PUT the ORIGIN "O", and alignment of axes is a matter OF judgement

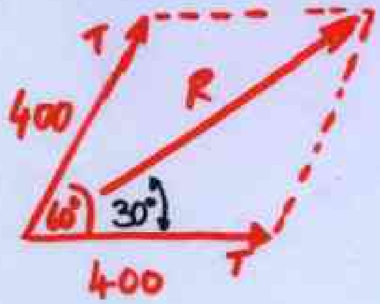
e.g.

Sample Problem 2/13 from M&K.



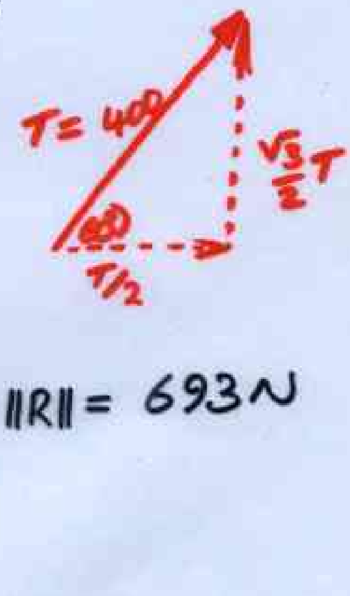
TENSIONS HAVE MAGNITUDE 400N
WHAT IS FORCE $\vec{R}$ exerted on pulley?

Again, more than one way to solve this ...



Parallelogram /graphical ...
IF YOU DRAW TO SCALE YOU can measure from DIAGRAM
$\|R\| \approx 693N$,
DIRECTION IS as SHOWN.

Components: use x-y axes as shown:



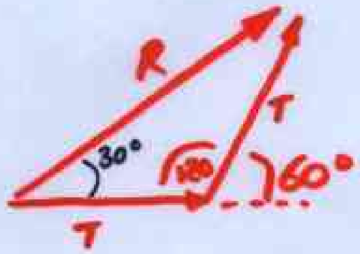
lower tension is EASY : $\boxed{400\hat{\imath}}$

UPPER one needs A LITTLE TRIG
$\Rightarrow \boxed{200\hat{\imath} + 200\cdot\sqrt{3}\,\hat{\jmath}}$

ADD to get $\vec{R} = 600\hat{\imath} + 346\hat{\jmath} \Rightarrow \|R\| = 693N$

TRIANGLE RULE / TRIG :

COSINE RULE:

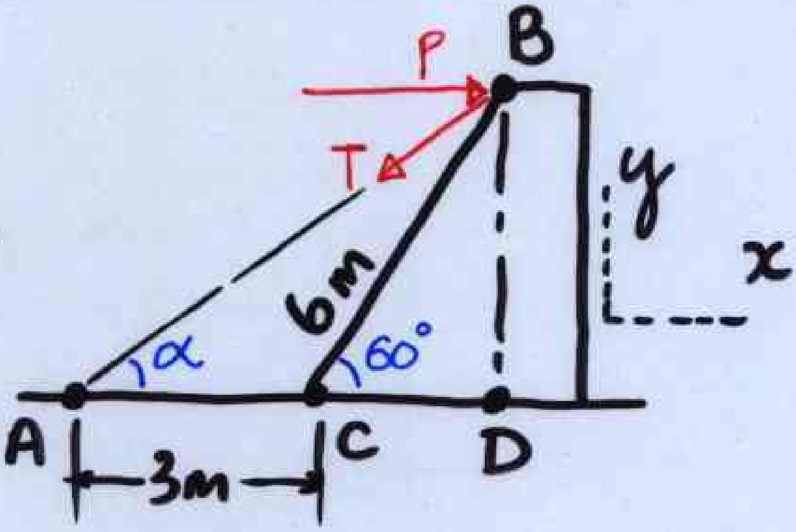


$$\|R\|^2 = \|T\|^2 + \|T\|^2 - 2\|T\|\|T\| \cos(120°)$$
$$= 1600 + 1600 - 2(400)(400)(-\tfrac{1}{2})$$
$$= 1600 + 1600 + 1600 = 4800$$
$$\Rightarrow \|R\| = \sqrt{4800} = 693N$$

DIRxn : 30° to HORIZONTAL

EXAMPLE

Combine the two forces, $P = 800\,N$ and $T = 600\,N$, into a single equivalent force, $\underset{\sim}{R}$.

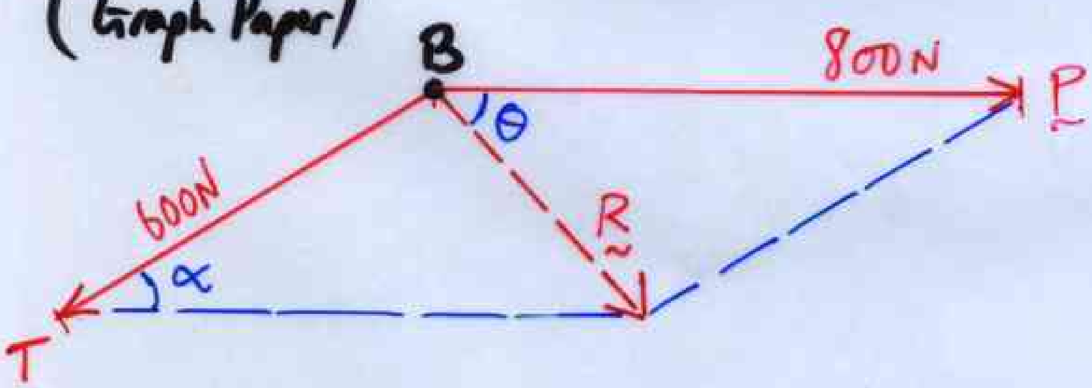


GRAPHICAL SOL^{N}

$$\tan\alpha = \frac{|BD|}{|AD|}$$

$$= \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} \Rightarrow \alpha = 40.9^\circ$$

(Graph Paper)

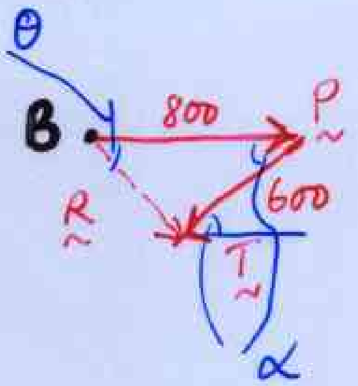


Measuring length and angle gives:

$$R \simeq \underline{525\,N}$$

$$\theta \simeq \underline{49^\circ}$$

GEOMETRY



Calculate α as before.

Cosine law gives:

$$R^2 = 600^2 + 800^2 - 2.600.800 \cos\alpha$$

$$\Rightarrow R = \underline{524\,N}$$

Sine law gives:

$$\frac{600}{\sin\theta} = \frac{524}{\sin\alpha} \Rightarrow \theta = \underline{48.6^\circ}$$

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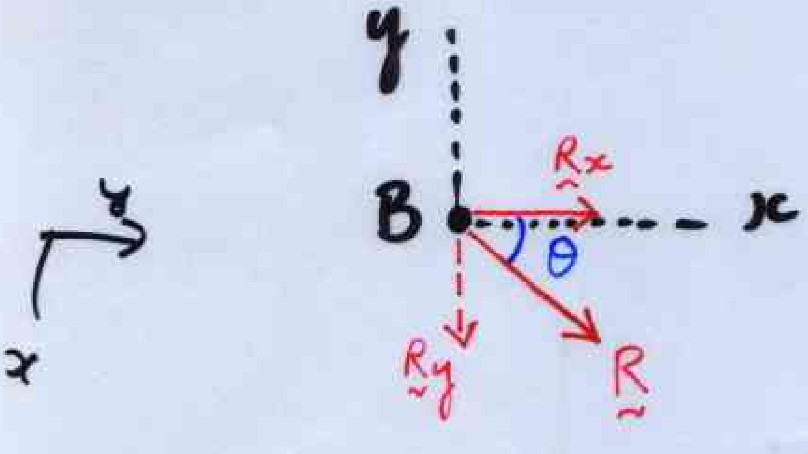
ALGEBRA

Choose x-y with origin B



$$R_x = \sum F_x = 800 - 600\cos\alpha$$
$$= 346\,\text{N}$$

$$R_y = \sum F_y = -600\sin\alpha = -393\,\text{N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{524\,\text{N}}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \underline{48.6^\circ}$$

A·9

Reference: ENGINEERING MECHANICS–VOL. 1 STATICS–2ND EDITION 1986
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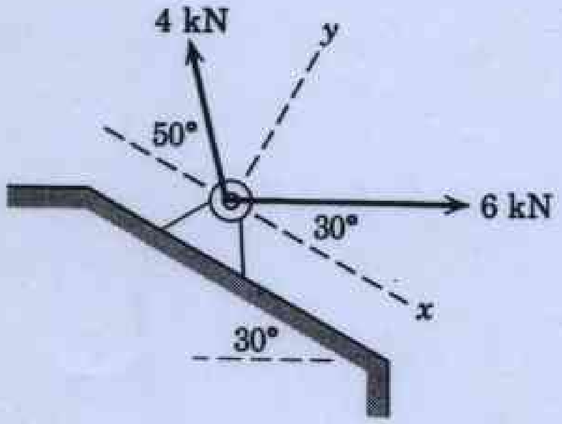
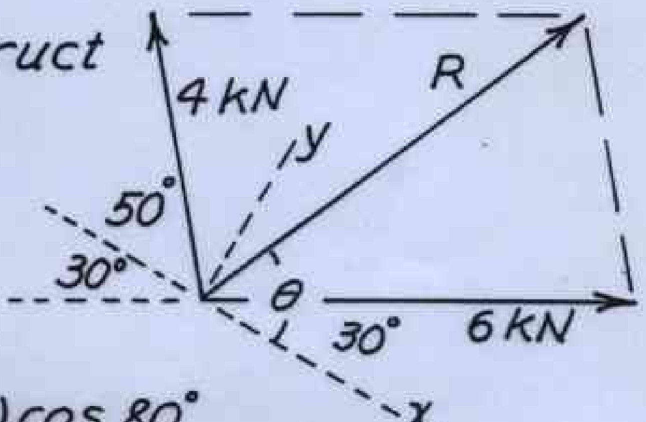
ART. 2/2 FORCE

Replace the two forces by a single equivalent force $\underline{R}$ and find the angle θ between $\underline{R}$ and the x-axis. Solve both geometrically and by using unit vectors $\underline{i}$ and $\underline{j}$.



Geometric

Graphical: construct parallelogram & measure R & θ.



Trigonometric:

Law of cosines

$R^2 = 4^2 + 6^2 - 2(4)(6)\cos 80°$

$= 43.7$, $\boxed{R = 6.61 \text{ kN}}$

$4^2 = (6.61)^2 + 6^2 - 2(6.61)(6)\cos(\theta - 30°)$

$\theta - 30° = \cos^{-1} 0.8029 = 36.6°$; $\boxed{\theta = 66.6°}$

Vector algebra

$R_x = 6\cos 30° - 4\cos 50° = 2.63 \text{ kN}$

$R_y = 6\sin 30° + 4\sin 50° = 6.06 \text{ kN}$

$\boxed{\underline{R} = 2.63\underline{i} + 6.06\underline{j} \text{ kN}}$, $\theta = \tan^{-1}\frac{6.06}{2.63} = \boxed{66.6°}$

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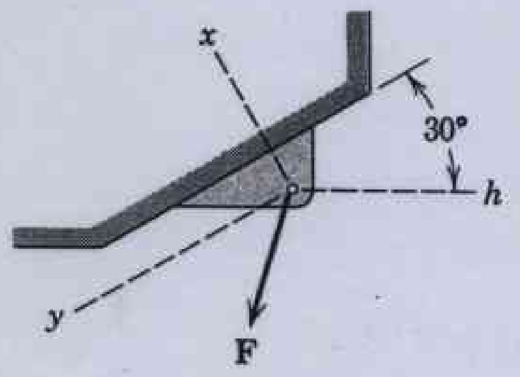
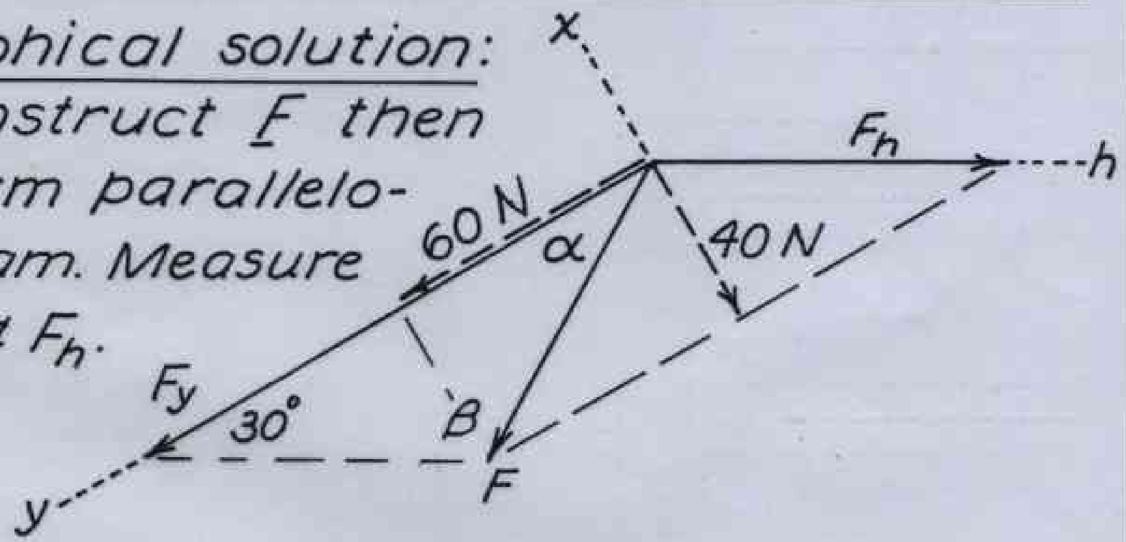
ART. 2/3 RECTANGULAR COMPONENTS (2-D)

Force $\underline{F}$ in rectangular components is given by $\underline{F} = -40\underline{i} + 60\underline{j}$ N. Determine the non-rectangular components of $\underline{F}$ in the y- and h-directions.



Graphical solution:

Construct $\underline{F}$ then form parallelogram. Measure F_y & F_h.



Trigonometric solution:

$\alpha = \tan^{-1}\frac{40}{60} = 33.7°$, $F = \sqrt{(40)^2 + (60)^2} = 72.1$ N

Law of sines $\frac{72.1}{\sin 30°} = \frac{F_h}{\sin 33.7°}$, $\boxed{F_h = 80.0\text{ N}}$

$\beta = 180 - 30 - 33.7 = 116.3°$

$$\frac{F_y}{\sin 116.3°} = \frac{72.1}{\sin 30°}, \quad \boxed{F_y = 129.3\text{ N}}$$

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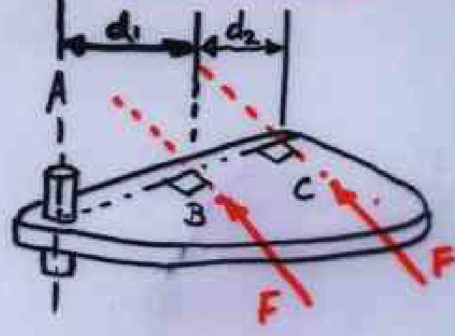
MOMENT:

A FORCE WILL TEND TO ACCELERATE BODY IN DIRECTION OF APPLICATION.

A SECOND EFFECT IS THAT IT WILL TEND TO ROTATE THE BODY. ABOUT AN AXIS



EVEN THOUGH THE 2 FORCES at B & C HAVE EQUAL MAGNITUDE, THEIR MOMENTS ABOUT AXIS A ARE DIFFERENT

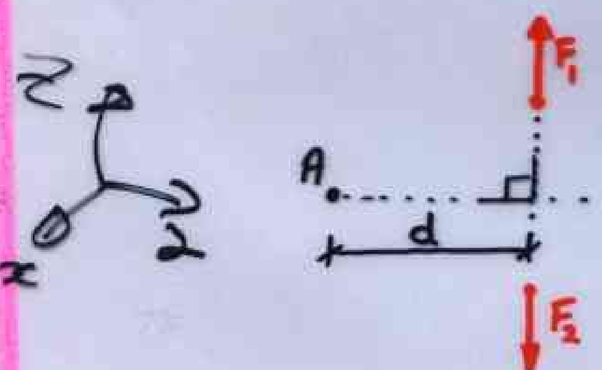
Magnitude of moment:

$M = Fd$, F is already familiar

"d" is the "MOMENT ARM":

PERPENDICULAR DISTANCE between axis and the line of action of force.

Easier to see in 2D (then axis is a point)



DIRECTION of FORCE is IMPORTANT

$\vec{F}_1$ would cancel moment of $\vec{F}_2$ about A

$\therefore$ we define moment as a VECTOR QUANTITY

$\|\vec{F}_1\| = \|\vec{F}_2\|$

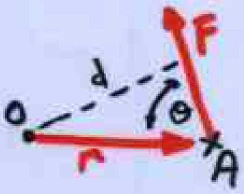
MOMENTS - Right HAND Rule

This is A CONVENTION to tell you the direction of moment vector. HARD TO DRAW

Leads ON TO USE OF CROSS-PRODUCT

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{r}$ is VECTOR from Axis O to point of application A



FOR 2D case $\|\vec{M}\| = \|\vec{r} \times \vec{F}\| = \|\vec{r}\| \|\vec{F}\| \sin\theta$

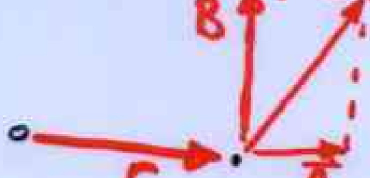
$= d \|F\|$ (since $d = \|\vec{r}\| \sin\theta$)

which is WHAT WE HAD BEFORE

However, this also APPLIES TO GENERAL 3D case

VARIGNON'S THEOREM

"moment of a force about a point is equal to sum of moments of components about that point"



Basically ... if $\vec{F} = \vec{A} + \vec{B}$

$\Rightarrow \vec{M}_o = \vec{r} \times \vec{F} = \vec{r} \times (\vec{A} + \vec{B})$

$= \vec{r} \times \vec{A} + \vec{r} \times \vec{B}$